**Quiz 5, ASM1**

Time allowed 90 mins

The objective is to use Gibbs sampling to generate random variables ($Y$) with pdf

$$f_Y(y) \propto \frac{ye^{-y}}{(5y+1)^2}, \quad y > 0$$

. We introduce a variable $Z$ such that the joint distribution of $Y, Z$ has pdf

$$f_{Y,Z}(y,z) \propto yze^{-5yz-y-z}, \quad y > 0, z > 0$$

The marginal distribution of $Y$ then agrees with the pdf $f_Y$ of interest.

1. Evaluate the constant of proportionality by numerically integrating the pdf $f_Y$ from 0 to 10 as a Riemann sum with interval size $10^{-4}$ and plot the density.

2. The conditional distribution of $Z$ given $Y = y$ is Gamma$(2, 5y + 1)$. Use symmetry to find the conditional distribution of $Y$ given $Z = z$.

3. Use these to set up a Gibbs sampling chain to generate $Y_2, Z_2, Y_3, \cdots, Y_N, Z_N$ for N=10000 starting with $Y_1, Z_1$ independent exponential(1) random variables.

4. Overlay the density in 1 with a kernel density estimator of $Y_1, \cdots, Y_N$.

5. Generate $Y_2, Z_2, Y_3, \cdots, Y_5$ using the Gibbs sampling chain and call $Y_5$ as $X_1$. Repeat with another random chain to obtain $X_2$. Proceed to generate N observations of $X$. Now add the kernel density estimator of $X_1, \cdots, X_N$ to the existing plot.