Quiz 4, ASM1
Time allowed 40 mins

The objective is to use Gibbs sampling to generate random variables (X, Y) jointly distributed as

$$\mathcal{N}_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4, 2\rho \\ 2\rho, 1 \end{pmatrix}\right).$$

The joint density is

$$\frac{1}{4\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{4} - \rho xy + y^2\right)\right\}$$

We start with independent random variables $X_1 \sim \mathcal{N}(0, 4)$ and $Y_1 \sim \mathcal{N}(0, 1)$

1. Find the conditional distribution of X given Y. We generate X_2 given Y_1 from this distribution.
2. Find the conditional distribution of Y given X. We generate Y_2 given X_2 from this distribution.
3. Find the joint distribution of Y_1, X_2, Y_2.
4. Integrate the answer in part 3 over Y_1 to obtain the joint distribution of (X_2, Y_2).