Indian Statistical Institute
B S D S Second Year
Second Semester, 2025-26
Final Examination
Advanced Statistical Methods I: Part II

25.05.26 **Total score 55** **Duration: 90 minutes**

The Maximum score that a student can get in the two parts combined is 100

Name **Student ID**

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1. Write your name and ID on each page.
2. Numbers in brackets denote total points allotted to each question.
3. You may use calculator.
4. Laptops and phones are not allowed.
5. You are allowed to bring two pages (one A4 sheet two sided OR two A4 sheets one sided) of notes.
6. Show all your work.

Qn no	1	2	3	4	Total
Marks	15	15	15	10	55
Obtained					

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1. (5+5+5=15) The following question refers to the Accept-Reject method with f being the target density and g being the proposal density. The algorithm accpts a proposal point Y if $U \leq \frac{1}{M}\frac{f(Y)}{g(Y)}$ where U is an independent standard uniform random variable.

   (a) Why do we need f to be absolutely continuous with respect to g?

   (b) Why is it okay if g is not absolutely continuous with respect to f?

   (c) Why do we need $M > f(x)/g(x)$ for all x?

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2. (7+8=15) Consider importance sampling where f is the target density, g is the candidate density and the objective is to estimate $\theta = \mathbb{E}_f(h(X))$. Suppose $(y_1, y_2, \cdots, y_n)$ are the random samples generated from g. Define

$$\hat{\theta}_g = \frac{1}{n} \sum_{i=1}^{n} \frac{f(y_i)}{g(y_i)} h(y_i)$$

(a) Show that $\hat{\theta}_g$ is unbiased for θ.

(b) Show that the variance of $\hat{\theta}_g$ is minimized when $g \propto f \mid h \mid$.

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3. (5+5+5=15) Let π be the pmf of a discrete distribution with $\Omega = \{x : \pi(x) > 0\}$ and cardinality of Ω is d. We want to generate samples from π. But π is known upto a constant, that is $\pi(x) = Cw(x)$ where w is known, but C is unknown. We define $p(x, y) = \min(\frac{w(y)}{w(x)}, 1)$ for $y \neq x$ and $p(x, x) = 1 - \sum_{y \neq x} p(x, y)$. Show the following

   (a) p is a transition matrix.

   (b) p satisfies the detailed balance equations.

   (c) π is the stationary distribution of p.

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4. (5+5) The following questions are from the presentations. Answer any two.
   (a) What are concordant and discordant pairs in the nonparametric estimation of correlation?
   (b) What is burn-in in a Metropolis algorithm and why is it required?
   (c) How is Gibbs sampling used in Bayesian inference?
   (d) What is bagging and how is it used in classification trees?
   (e) Define value at risk and explain how it is reated to quantile estimation.
   (f) For generating observations from a $\text{Multinomial}_k(n, p)$, if n is small and k is large, what method will you use? What method will you use if n is large and k is small?

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