QUANTITATIVE SPATIAL REASONING
Spatial Reasoning

- Strategically important set(s)
- Directional spatial relationship
- Spatial Significance Index of zones
III.II.I. Strategically significant state(s)

\[
\begin{align*}
H/P(A_{ij}) &= -\sum_{i,j, i\neq j} \Pr[P(A_{ij}) \log \Pr[P(A_{ij})]] \\
H/C(A_{ij}) &= -\sum_{i,j, i\neq j} \Pr[C(A_{ij}) \log \Pr[C(A_{ij})]] \\
H/d(A_{ij}) &= -\sum_{i,j, i\neq j} \Pr[d(A_{ij}) \log \Pr[d(A_{ij})]] \\
H/d(A_{ji}) &= -\sum_{i,j, i\neq j} \Pr[d(A_{ji}) \log \Pr[d(A_{ji})]] \\
(SH_i)^p &= \min \left\{ H/P(A_{ij}) \right\} \\
(SH_i)^d &= \min \left\{ \min \left\{ H/d(A_{ij}), H/d(A_{ji}) \right\} \right\} \\
(SH_i)^c &= \min \left\{ H/C(A_{ij}) \right\} \\
(SA_i)^p &= \max \left\{ \sum_i NP(A_{ij}) \right\} \\
(SA_i)^d &= \min_{i,j} \left\{ \min \left[ \sum_i Nd(A_{ij}), \sum_j Nd(A_{ji}) \right] \right\} \\
(SA_i)^c &= \max \left\{ \sum_i C(A_{ij}) \right\}
\end{align*}
\]

\[
P(A_{ij}) = P[(A_i \oplus B) \cap (A_j)]
\]

\[
d(A_{ij}) = \min_{i \neq j} \left\{ n : A_j \subseteq (A_i \oplus nB) \right\}
\]

\[
C(A_{ij}) = C(A_{ji})
\]

\[
\begin{bmatrix}
    \rho(A_{ij}) \\
    \max \left[ d(A_{ij}), d(A_{ji}) \right]
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \rho\left( N\Delta(A_{ij}) \right) \\
    \max \left[ N\Delta(A_{ij}), N\Delta(A_{ji}) \right]
\end{bmatrix}
\]
Matrices and Parameters

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24 March 2013
B. S. Daya Sagar
Strategically significant state(s) w.r.t 10 parameters
Directional Spatial Relationship

http://www.isibang.ac.in/~bsdsagar/AnimationOfDirectionalSpatialRelationship.wmv

Fig. 1. (A1, A2, A3) three disjoint objects possessing different directional spatial relationship.

Fig. 4 Shows (a) origins of structuring element, and their corresponding directions in (b) and color codes in (c).

Fig. 3. Structuring element is shown with different possible origins. Except the first structuring element for which the origin is shown at the center, all other eight structuring elements are with other eight possible positions as origins. These eight other structuring elements are symmetric structuring elements as their transposes are not equal to their non-transposed versions.

\[
\Delta(A_i, A_j) = i \min_{\forall i} n : A_j \subseteq \left( A_i \oplus n B_i \right)
\]

TABLE 1. DISTANCES, UNIQUE ORIGINS AND DIRECTIONS

<table>
<thead>
<tr>
<th>Minimum Dilation Distances</th>
<th>Unique Origins</th>
<th>Directional Relations</th>
<th>Visualization of Directional Relations</th>
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24 March 2013

B. S. Daya Sagar
Directional Spatial Relationship
DETECTION
OF SPATIALLY
SIGNIFICANT ZONES
The ability to derive spatially significant zones within a cluster of zones has interesting applications in understanding commonly sharing physical mechanisms. Using morphological dilation distance technique, we introduce geometrically-based criteria that serve as indicator of the spatial significance of zones within a cluster of zones. This presentation focuses on the problem of identifying zones that are ‘strategic’ in the sense that they are the most central or important based on their proximity to other zones. We have applied this technique to a task aiming at detecting spatially significant water body from a cluster of water bodies retrieved from IRS LISS-III multispectral satellite data.
INTRODUCTION

SPATIALLY SIGNIFICANT ZONES
Spatial Entities can be well identified/ mapped from Digital Elevation Models generated from high resolution remotely sensed data.

- Spatial Entities – water bodies, zones of influence, geomorphic basins, and urban features of the specific thematic maps.

Understanding the organization of these spatial entities is an important aspect from the point of ‘Spatial Reasoning’.
Spatially Significant Zone (SSZ) can be defined as “a zone from which it is easy to reach all of its neighbouring zones”.

SSZ necessarily be at a strategic location, and also possessing relatively larger size.

Cluster of spatial entities (zones) can be treated as a ‘Spatial System’.

- **Eg:** Geomorphological basin (cluster of sub-basins) consists of sub-basins (zones), and sub-basins consist of still minor sub-basins, and so on.
• SSZ within a cluster of zones possess a geometric characteristics that is greater proximity to other zones.
• Identifying the spatial significance of a zone from geometric point of view based on qualitative spatial reasoning is non-trivial.
• Recognizing SSZ within a spatial system composed of various zones could be accomplished quantitatively.
  ◦ Need to define an appropriate measure of the spatially significance of a zone.
For a geometric basin \((A_i)\), if \(A_1\) is considered as an origin zone, then all the other zones \((A_2-A_{10})\) are treated as destination zones.
To provide an equation based on dilation distances among zones in a cluster.

To automatically compute spatial significance index (SSI) for each zone of a cluster of zones.

- Morphological Dilation
- Spatial System and its Subsystems
- Dilation Distances Between Origin and Destination Zones
- Spatial Significance Index of a Zone
Binary dilation is a fundamental morphological operation, can be performed on any set on 2-D Euclidean space.

Dilation: The Boolean OR transformation of a set A by a set B.

\[ A \oplus B = \left\{ a : B_a \cap A \neq \emptyset \right\} = \bigcup_{b \in B} A_b \]

- **Multiscale Dilations** can be performed by varying the size of the structuring element \((nB)\). \((n \geq 0)\)

- Iterative dilations can also be represented mathematically, as follows:

\[ (A \oplus nB) = (A \oplus B) \oplus B \oplus \cdots \oplus B \]

\[ n = 0, 1, 2, \ldots, N \]
Let ‘A’ be a cluster of zones composed of a number of non-empty, compact sets (zones) \( A_1, A_2, A_3, \ldots, A_N \)

Any pair of zones \( A_i \) & \( A_j \), from this cluster, that \( i \neq j \), the following spatial relations holds true:

\[ A_i \cap A_j = \emptyset \]

- **The relations I & II** would be satisfied for the cases of water bodies, nodes, point-specific data.
- **Relation III** will be satisfied, if all the zones of a cluster are in contiguous form.
Based on Euclidean metric, determining distances between spatial objects is a challenge.

If the sizes of the zones are similar, simple Euclidean distances would suffice to detect the spatially significant centroid of a corresponding zone.

If the cluster consists of dissimilar shapes and sizes, then detecting the spatially significant zone can be done through –

- Computation of zone centroids
  - Minimal Skeleton Points (MSPs)
- Euclidean distance between centroids of two zones
  - Zones Morphological properties cannot be considered

ITERATIVE DILATION DISTANCE IS A BETTER CHOICE TO COMPUTE DISTANCES BETWEEN ZONES
Iterative Dilation

Distances

Let non-empty, disjoint compact zones \( A_i \) and \( A_j \) be the original and destination zones. \((A_i < A_j)\)

- The distance from \( A_i \) to \( A_j \) represented by:
  \[
  d\left( A_{ji} \right) = \min_{i \neq j} \left( n : A_i \subseteq (A_j \oplus nB) \right)
  \]

- The distance between \( A_j \) and \( A_i \) represented by:
  \[
  d\left( A_{ij} \right) = \min_{i \neq j} \left( n : A_j \subseteq (A_i \oplus nB) \right)
  \]

The following conditions will be satisfied, iff both ‘\( A_i \)’ & ‘\( A_j \)’ possess identical size, shape & orientation.

\[
\begin{align*}
  d\left( A_{ij} \right) &= d\left( A_{ji} \right) \\
  d\left( A_{ii} \right) &= 0, \quad d\left( A_{ij} \right) &\neq d\left( A_{ji} \right)
\end{align*}
\]

\[
\text{dilation distances } d\left( A_{ij} \right) = 11, \text{ and } d\left( A_{ji} \right) = 7, \text{ and } \rho\left( A_{ij} \right) = 7
\]
If the compact zones shape-sizes are dissimilar, then:

\[ d(A_{ij}) \neq d(A_{ji}) \]

- The min. of \( d(A_{ij}) \) and \( d(A_{ji}) \) is Hausdorff dilation distance:
  \[ \rho(A_{ij}) = \min \left( d : d(A_{ij}), d(A_{ji}) \right) \]

- The max. distance (\( d_{\text{max}} \)) between origin zone \((A_i)\) & destination zone \((A_j)\) is computed as:
  \[ d_{\text{max}}(A_{ij}) = \max \forall j \left( \min \left( n : (A_j \subseteq (A_i \oplus nB)) \right) \right) = \min \left\{ n : \bigcup_{j=1}^{N} A_j \subseteq (A_i \oplus nB) \right\} \]

- \( d_{\text{max}} \) between the destination zones and an origin zone is computed as:
  \[ d_{\text{max}}(A_{ji}) = \max \forall j \left( \min \left( n : (A_i \subseteq (A_j \oplus nB)) \right) \right) \]

Estimation of the dilation distance between the origin & destination zones is justified as such as a **dilation distance is essential to compute distances between zones.**

**Limitation:** This distance is essential affected by the object’s boundary points that are farthest out with respect to other spatial objects.
A zone \((A_i)\) is said to the best zone and termed as spatially the most important zone, if it satisfies the below characteristics:

- If it is located in a place closer to all \(A_j\)s, and
- Reaching \(A_i\) from all \(A_j\)s required shorter distance.

Spatial Significance Index of a zone is defined as involving dilation-distances between origin \((A_i)\) and destination zones \((A_j)\).

\[
SSI = \min_{\forall i} \left( d_{\max} \left( A_{ij} \right) \right)
\]

SSI of a zone is a dimensionless unit.

Lower the SSI of a zone \((A_i)\) in a cluster of zones, the higher is its significance.
Normalized Spatial Significance Index (NSSI) that ranges between 0 and 1 takes form of:

\[
NSSI = \frac{\min_{\forall i} (d_{\max}(A_{ij}))}{\max_{\forall i} (d_{\max}(A_{ij}))}
\]

- If the zones of a cluster are **identical**, then:
  \[
  \min_{\forall i} (d_{\max}(A_{ij})) = \min_{\forall j} (d_{\max}(A_{ji}))
  \]

- If the zones of a cluster are **dissimilar**, then:
  \[
  \min_{\forall i} (d_{\max}(A_{ij})) \neq \min_{\forall j} (d_{\max}(A_{ji}))
  \]

- When all the zones in a cluster are similar both in terms of size & shape, the following relationship holds **good**.
- Small water bodies and their zones of influence of varied sizes and shapes arranged heterogeneously.
- Max. dilation distances observed from distances computed between every water body and every other water body belonging to a cluster of 66 water bodies.
- The observed min. distances among 66 max. distances for both water bodies & zones include 53 & 52 respectively.
- The max. distances among 66 max. distances for both water bodies and zones observed include 109 & 110 respectively.

a. LISS-III input image.

b. Extracted water bodies from RS data. (60)
c. Zones of influence, by corresponding water bodies. (66)
d. Water bodies and zones with labeling.
### EXPERIMENTAL RESULTS

**SSI – Water Bodies Zones of Influence**

**Graphical Representations:**

(a) 
![Graph 1](image)

(b) 
![Graph 2](image)

**Table I: SSI of Top Five Water Bodies and Zones**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Water Body (W)</th>
<th>D-Dist</th>
<th>Zone (Z)</th>
<th>D-Dist</th>
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<th>NSSI (Z)</th>
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The computational complexity increases with increasing:
1. No. of spatial objects
2. Spatial resolution

Note: The no. of dilation distances required to be computed increases with no. of spatial objects, their sizes of the individual spatial objects.
Conclusions

- This iterative dilation distances technique can be extended:
  - To a wide class of metric spaces and to other representations (objects bounded by 2-D vectors), and
  - To 3-D case by replacing dilation distance with gray-scale geodesic distances.

- This technique useful insights in:
  i. Clustering-classification frameworks,
  ii. Detecting the spatially significant segmented zones obtained via various segmentation approaches,
  iii. Automatically deriving a central node from a large no. of nodes,
  iv. Determining the influence of a node in a vector-based network setting,
  v. Deciding on nodal centre(s) to establish an administrative facility, from a cluster of cadastral zones from mapped from remotely sensed satellite data.
I would like to gratefully acknowledge Robert Marschallinger, Fritz Zobl, Vera Pawlowsky-Glahn, Qiuming Cheng, Gina Ross, Katsuaki Koike and Jean Serra. Support given by Bimal Roy and Sankar Pal (Current and Former Directors of Indian Statistical Institute) who created a great environment for academic research is gratefully acknowledged.


Selected References


Thank You

Q & A
Thank You