

# Recent Advances in Operator Theory and Operator Algebras

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**Workshop Part**  
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## **Titles and Abstracts**

**Hari Bercovici**

*Title: Schubert calculus and operator theory*

**Abstract:** Schubert calculus is the intersection theory associated with Grassmann manifolds. It answers enumeration questions such as the following one: given four lines in three-dimensional space, how many lines intersect all four? (Assume the lines are in "general position".) The answers to these questions can be obtained algorithmically using the Littlewood-Richardson rule. The same rule is seen to appear in the classification of the invariant subspaces of certain operators, and it is also useful in characterizing the eigenvalues of sums of matrices or the singular values of products. In these lectures we

will introduce the basic facts of intersection theory and show how they are applied in various questions of operator theory.

## David Kerr

*Title: Amenability, soficity, entropy, and operator algebras*

**Abstract:** The notion of amenability has long played a fundamental role in the study of both operator algebras and locally compact groups. The internal kind of finite approximation that it represents leads both to an elaborate structure theory in many cases and to the formulation of numerical invariants like entropy. Amenability admits an external counterpart for groups in the form of soficity, the idea of which was introduced by Gromov in the late 1990s in the framework of Gottschalk's surjunctivity conjecture. While the much weaker concept of soficity cannot be expected to provide any kind of detailed structural picture, it does lend itself to the definition of an invariant like entropy, as Bowen discovered in a remarkable breakthrough a few years ago. One outcome of Bowen's work is a far-reaching generalization of the Ornstein-Weiss classification of Bernoulli actions over countably infinite amenable groups.

I will discuss both amenability and soficity for groups, their relationship to operator algebras, and their role in the theory of dynamical entropy. In particular I will discuss the Fuglede-Kadison determinant in group von Neumann algebras as it arises in the calculation of entropy for algebraic actions.

## Elias Katsoulis

*Title: Non-Selfadjoint Operator Algebras: dynamics, classification and  $C^*$ -envelopes*

**Abstract:** The goal of these lectures is twofold. The first goal is to offer an introduction to non-selfadjoint operator algebra theory keeping in perspective the historical development of the subject. Starting with the nest and CSL algebras we will move forward towards more modern algebras, including semicrossed products of  $C^*$ -algebras, graph algebras and their generalizations, tensor algebras of  $C^*$ -correspondences. The central results of each theory will be reviewed and in certain cases we will sketch proofs.

The other goal of the lectures is to demonstrate the delicate interplay

between dynamics, dilation theory and the classification of certain non-selfadjoint operator algebras. These ideas originate in the work of Arveson in the sixties and recently there has been a good deal of activity in that subject. I will introduce the  $C^*$ -envelope of an operator algebra and review some the general theory (existence, maximal dilations, boundary representations). In certain concrete cases, the  $C^*$ -envelope will be calculated explicitly. In particular, I will explain how to build various operator algebras from multivariable dynamical systems and calculate, whenever possible, their  $C^*$ -envelopes. For these multivariable algebras I will also offer various classification schemes, which in certain cases are complete.

### **Dan Timotin**

Title: *Stable polynomials and the solution to the Kadison-Singer conjecture*  
Short syllabus:

- Stable polynomials: some results of Borcea and Branden
- Interlacing families of polynomials
- The Kadison-Singer conjecture and some reformulations
- The mixed characteristic polynomial
- Estimates on the roots of the mixed characteristic polynomials
- Discrepancy theory - Solution to the Kadison-Singer conjecture