Noncommutative Mathematics and Applications. 24-26 October 2024

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Titles and Abstracts

1. B V Rajarama Bhat

Title: *Joint and outer spectral radii*

Abstract: The notions of joint and outer spectral radii are extended to the setting of Hilbert C*-bimodules. A Rota-Strang type characterisation is proved for the joint spectral radius. In this general setting, an approximation result for the joint spectral radius in terms of the outer spectral radius has been established. This work leads to a new proof of the Wielandt-Friedland's formula for the spectral radius of positive maps. It is observed that algebras generated by tuples of matrices can be determined and their dimensions can be computed by realizing them as linear span of Choi-Kraus coefficients of some easily computable completely positive maps. This talk is based on a joint work with Biswarup Saha and Prajakta Sahasrabuddhe.

2. Tirthankar Bhattacharyya

Title: *Gromov-Hausdorff convergence of families of CP maps*

Abstract: Connes and van Suijlekom introduced spectral truncations in [Alain Connes and Walter D. van Suijlekom, Spectral truncations in noncommutative geometry and operator systems. Comm. Math. Phys.383(2021), no.3, 2021–2067]. We use their framework to prove Gromov-Hausdorff convergence of families of CP maps from the truncated systems. We illustrate it with examples. The talk will develop all necessary material from scratch. This is joint work with Ritul Duhan and Chandan Pradhan.

3. Jyotishman Bhowmick

Title: *Levi-Civita connections on a class of quantum homogeneous spaces*

Abstract: A noncommutative manifold structure on a $*$-algebra B is the prescription of a differential graded algebra $(\Omega^\bullet, \wedge, d)$, called a differential calculus on B. When B is the commutative $*$-algebra of smooth functions on a function M, the algebra of differential forms on M provides the natural example. In the noncommutative case, a wealth of examples of such differential calculi on quantum groups and their homogeneous spaces have been studied in the literature.

Thus, given a differential calculus $(\Omega^\bullet, \wedge, d)$, the notion of (Riemannian) metrics and connections on the space of one-forms makes sense. In this talk, we will try to address the following question:

Given a differential calculus $(\Omega^\bullet, \wedge, d)$ on a $*$-algebra B and a metric on Ω^1, does there exist a unique connection on Ω^1 which is torsionless and compatible with the given metric?

We will show that for the differential calculi constructed by Heckenberger-Kolb on the (quantized) function algebra of irreducible flag manifolds, the answer is yes. This is based on a joint work with B. Ghosh, A. Krutov and R.O. Buachalla.

4. Indranil Biswas

Title: *A canonical connection on bundles on Riemann surfaces and Quillen connection on the theta bundle*
Abstract: We investigate the differential geometric aspects of some canonical torsors on the moduli space of vector bundles on a compact Riemann surface. (Joint work with Jacques Hurtubise.)

5. Arup Chattopadhyay

Title: *Spectral shift functions of all orders*
Abstract: Let $n \in \mathbb{N}$ and let H_0, V be self-adjoint operators such that V is bounded and $V(H_0 - i)^{-p} \in \mathcal{S}^{n/p}$ (Schatten -n/p class) for $p = 1, \ldots, n$. In this talk, I will discuss the existence, uniqueness up to polynomial summands, and regularity properties of all higher order spectral shift functions associated to the perturbation theory of $t \mapsto H_0 + tV$ ($t \in [0, 1]$). This is a joint work with Teun D.H. van Nuland and Chandan Pradhan.

6. Probal Chaudhuri

Title: *On Rao-Blackwell Theorem and Cramer-Rao Inequality*
Abstract: During 2020-2022, when we were all confined to home due to the Covid pandemic, I and Prof. Kalyan Sinha had several long phone conversations discussing various academic and nonacademic issues. One specific topic that Prof. Sinha was interested in was to develop analogs of Rao-Blackwell and Cramer-Rao type results in Quantum Probability. This gave me an excellent opportunity to revive my interest in statistical decision theory. We intensely discussed concepts of likelihood and sufficiency and their fundamental roles in statistical decision theory in classical probability setting. Some of the interesting ideas that emerged from those discussions as well as some ideas that resurfaced in my mind from the remote past, when I was doing my postgraduate studies, I will share in this talk.

7. Sameer Chavan

Title: *Wold-type decomposition for Weighted shifts on rootless directed trees*
Abstract: It is known that a weighted shift on a rootless directed tree may have a nontrivial nonanalytic part. Under some mild assumptions, it is possible to identify it with a bilateral weighted shift. Moreover, reducibility of the hyper-range is closely related to the notion of the "balanced" shift. This is a joint work with Shailesh Trivedi

8. Satyajit Guin

Title: *A naive attempt to "Two Subfactors"—few curious and interesting results*
Abstract: Jones proposed the study of "two subfactors" of a II_1 factor as a quantization of two closed subspaces in a Hilbert space. In the absence of a general theory for two subfactors, it is imperative that one starts investigating some particular important class of two subfactors to gain insights. Motivated by this goal, we have initiated a systematic

study of a special class of two subfactors, namely a pair of spin model subfactors. In this talk, we discuss a few curious and interesting results in this direction. These are part of a joint work with Keshab Chandra Bakshi.

9. Sk Asfaq Hossain

Title: *Quantum symmetry in finite multigraphs*
Abstract: Quantum symmetry in simple graphs is a well-explored topic within noncommutative mathematics, and it has gained increasing significance in quantum information theory in recent years. In this talk, we will introduce various notions of quantum symmetry in finite multigraphs in the category of compact quantum groups, naturally extending the well-known constructions in simple graphs.

10. Soumalya Joardar

Title: *Compact quantum group actions on graph C^*-algebras*
Abstract: The class of graph C^*-algebras is a very important class of C^*-algebras. It is quite tractable due to the presence of its underlying combinatorial graph structure. Being a very important class of C^*-algebras, it is a very natural problem to understand its generalized symmetry modelled by compact quantum group actions. In this talk, I shall present an overview of compact quantum group actions on graph C^*-algebras.

11. Apoorva Khare

Title: *Beyond positivity preservers: Schoenberg's theorem for matrices with negative eigenvalues*
Abstract: Pólya and Szegő (1925) explained how the Schur product theorem (1911) implies: convergent power series with nonnegative coefficients preserve the class of (real symmetric) positive semidefinite matrices, when applied entrywise. A celebrated result of Schoenberg (1942) and Rudin (1959) proves the converse: there are no other such preservers. Many subsequent variants and extensions exist.

We extend this "Schur-Schoenberg characterization" to symmetric/Hermitian matrices, constrained by Negative Inertia (N.I.), i.e. the number of negative eigenvalues. Our main result classifies the entrywise functions of several real (or several complex) variables, that send tuples of matrices with N.I.s at most $(k_1, \ldots, k_m)$ to matrices with N.I. at most l – for any $k_1, \ldots, k_m, l \geq 0$. This extends Schoenberg's (multivariate) theorem where all k_i and l equal 0.

The proofs use a recently studied inflation operator on matrices, classical results by Sylvester and by Weyl, and Sidon sets (studied in number theory and combinatorics by Erdős–Turán and Bose–Chowla). As an application, we obtain the preservers of matrix inertia. (Joint with Alexander Belton, Dominique Guillot, and Mihai Putinar.)

12. Krishna Maddaly

Title: *Dynamical delocalisation in Point interaction models of random Schrodinger operators*
Abstract: A point interaction with one center of interaction is a model that informally has infinite potential at one point. We consider a random infinite center point interaction model and show that the time average upto time T of the time evolved position operator

grows with time. We explain the significance of such a result. This is joint work with Peter Hislop and Werner Kirsch.

13. Gadadhar Misra

Title: *Homogeneous analytic Hilbert modules – the case of the symmetrized bidisc*
Abstract: This talk is in two parts. In the first part, we discuss analytic Hilbert modules H over the polynomial ring consisting of holomorphic functions defined on a G- space X. We investigate the question of which of these are homogeneous under the natural action of the group G lifted to the algebra of bounded linear operators on H. In a slight departure from the past studies of such questions, here we don't assume transitivity of the group action. The main result is that the unitary invariants, like the curvature and the reproducing kernel of a homogeneous analytic Hilbert module, are now determined from their restriction to what we call a fundamental set Z for the group action. In case the group action is transitive, Z is a singleton. In the second half of the talk, we describe examples of homogeneous analytic Hilbert modules based on the symmetrized bi-disc under its bi-holomorphic automorphism group.

14. N. Namboodiri

Title: *Algebraic structure of regular probability measures on Hausdorff, locally compact topological groups*
Abstract: Let G be a locally compact Hausdorff group, and $P(G)$ denote the class of all regular probability measures on G. It is well known that $P(G)$ is a semi-group under the convolution of measures. It is also well known that the only ivertible elenents are Dirac delta measures. Analogously, we address the question of generalized invertibility of elements in $P(G)$.

15. Suchetana Samadder

Title: *Discrete Quantum Group of Outer Automorphisms of von Neumann algebras*
Abstract: The goal of this talk is to formulate a quantum group analogue of the group of outer automorphisms of von Neumann algebras equipped with a finite, faithful normal trace.

This will be done in the following two steps:

(1) Let $\mathcal{A}$ be an von Neumann algebra. Following the ideas due to S. Vaes, we construct a von Neumann algebraic discrete quantum group (DQG) $\mathcal{Q}(\mathcal{A})$ as the universal object in the category of DQGs coacting on $\mathcal{A}$.
(2) Moreover, if $\mathcal{A}$ is also equipped with a finite faithful normal trace τ, we construct a discrete quantum subgroup $\mathcal{Q}^{\tau,kac}(\mathcal{A})$ which is an universal for the τ-preserving coactions by a Kac type DQG.
Then we construct the discrete quantum group of outer automorphisms of $\mathcal{A}$, $\mathcal{Q}^{\tau,kac}_{out}$, by quotienting $\mathcal{Q}^{\tau,kac}(\mathcal{A})$ by a suitable quantum normal subgroup $\mathcal{Q}_{inn}(\mathcal{A})$ corresponding to what we call as 'inner' objects.

We will also try to discuss the discrete quantum outer automorphism groups for few concrete examples of finite dimensional von Neumann algebras.
This is based on an ongoing joint work with my supervisor D. Goswami.

16. Lingaraj Sahu

Title: *Perturbation of embedded eigenvalue and spectral concentration*
Abstract: Let T be a self-adjoint operator on a Hilbert space $\mathcal{H}$ with an eigenvalue λ embedded in the continuous spectrum. Aim of this talk is to discuss how the spectrum of the perturbed operator $T(V) = T + V$ varies with V. In particular we will analyses the persistence of embedded eigenvalue, disappearance of embedded eigenvalue and spectral concentration phenomena. To understand this better we will look at a simple model with finite rank perturbations V and show spectral concentration for a simple eigenvalue. If time permits we will discuss the case of degenerate eigenvalues. This work is jointly with Alok Marharana, Hemant Bansal and K B Sinha.

17. Ritabrata Sengupta

Title: *The likelihood operator and Fisher information in quantum probability*
Abstract: We study the problem of Quantum Likelihood Estimator (QLE) operators and their connection with quantum Fisher information (QFI). We revisit the standard procedure in this direction in classical theory and adapt it to the quantum case. It is observed that the present approaches to this problem tacitly assumes commutativity of one parameter density matrix $\rho(\theta)$ and its derivative, which, in general, need not be true.

18. Aryaman Sensarma

Title: *Russo-Dye type Theorem, Stinespring representation, and Radon Nikodým dervative for invariant block multilinear completely positive maps*
Abstract: In this talk, we explore fundamental properties of invariant multilinear completely positive (CP) maps. In particular, we establish a Russo-Dye type theorem for invariant multilinear positive maps on both commutative C^*-algebras and finite-dimensional C^*-algebras. We show that every invariant multilinear CP map is automatically symmetric and completely bounded.

Furthermore, we introduce a multilinear version of the invariant block CP map $\varphi = [\varphi_{ij}] : M_n(\mathcal{A})^k \to M_n(\mathcal{B}(\mathcal{H}))$ and show that each φ_{ij} can be dilated to a common commutative tuple of $*$-homomorphisms. A special case of this result refines the Stinespring type dilation theorems of J. Heo and A. Kaplan.

As an application, we also derive a Russo-Dye type theorem for invariant multilinear completely positive maps. Finally, using minimal Stinespring dilation we obtain Radon Nikodým theorem in this setup. We will also discuss several open questions in this direction. This work is a joint collaboration with Dr. Anindya Ghatak.

19. K. B. Sinha

Title: *My mathematical journey; some open questions*
Abstract: TBA