# MATHEMATICAL MORPHOLOGY IN GEOSCIENCES AND GISci


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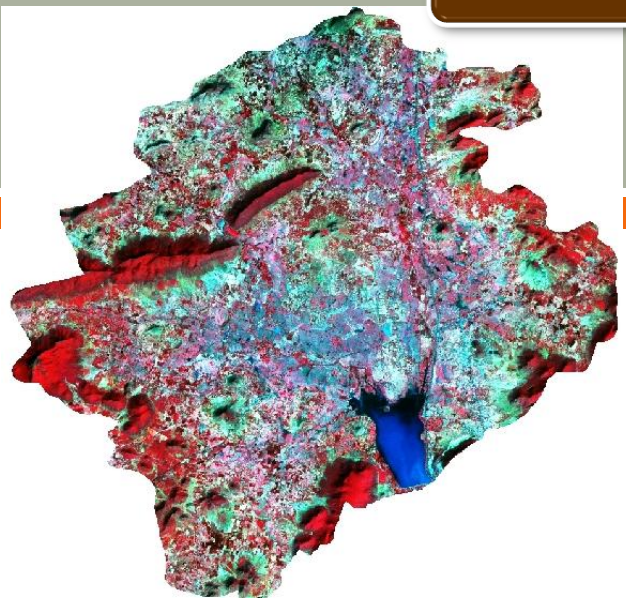
Systems Science and Informatics Unit (SSIU)

29th September 2014

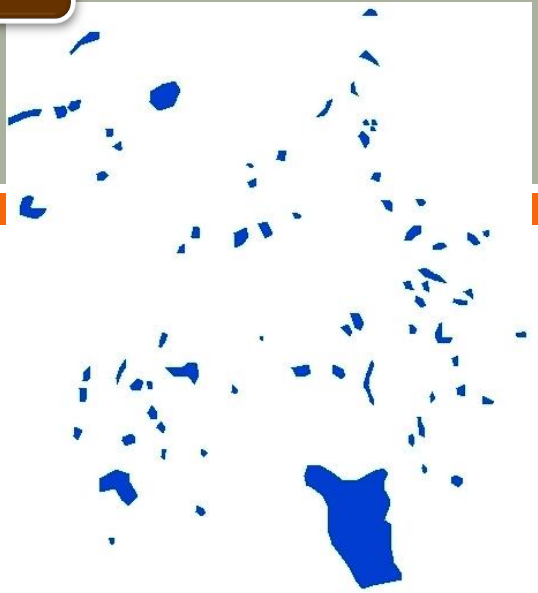
Centre for Earth & Space Sciences, UoH, Hyderabad

# OVERALL METHODOLOGY



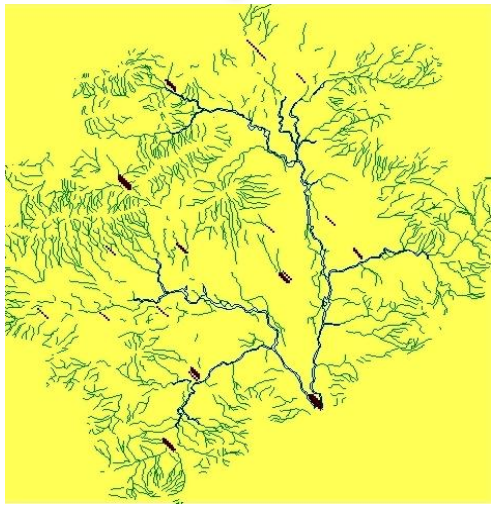
Remote Sensing Data

Segmentation

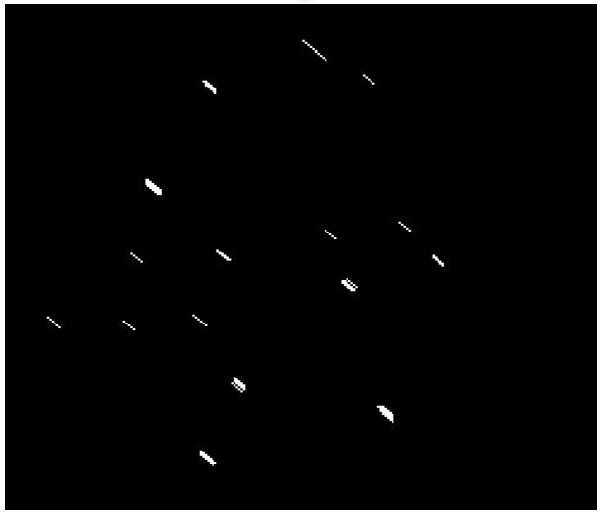


Extraction of Water bodies from RS Data

## Pattern Spectrum



## Recursive Openings



## 1-D Structuring Elements

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

| | | |
|---|---|---|
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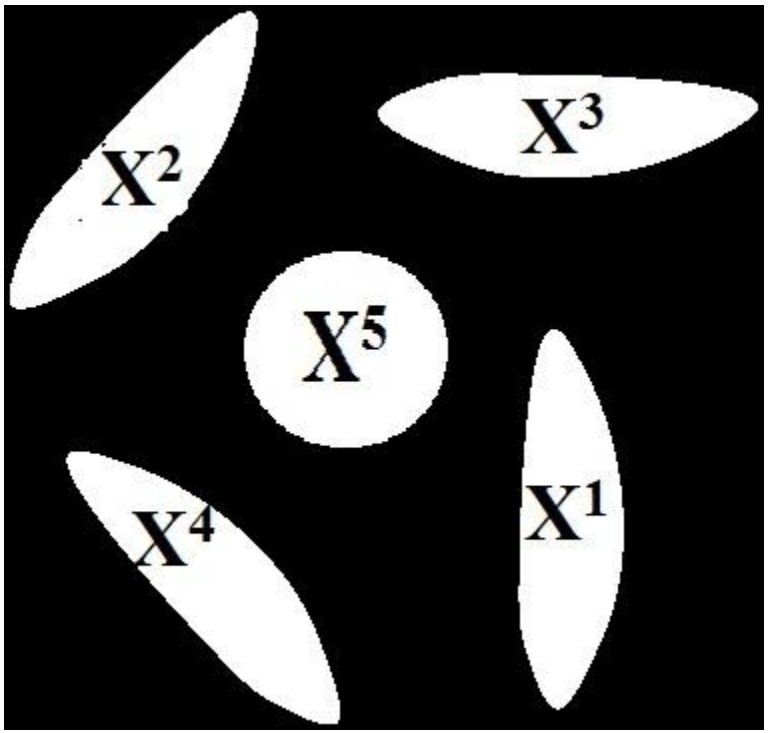
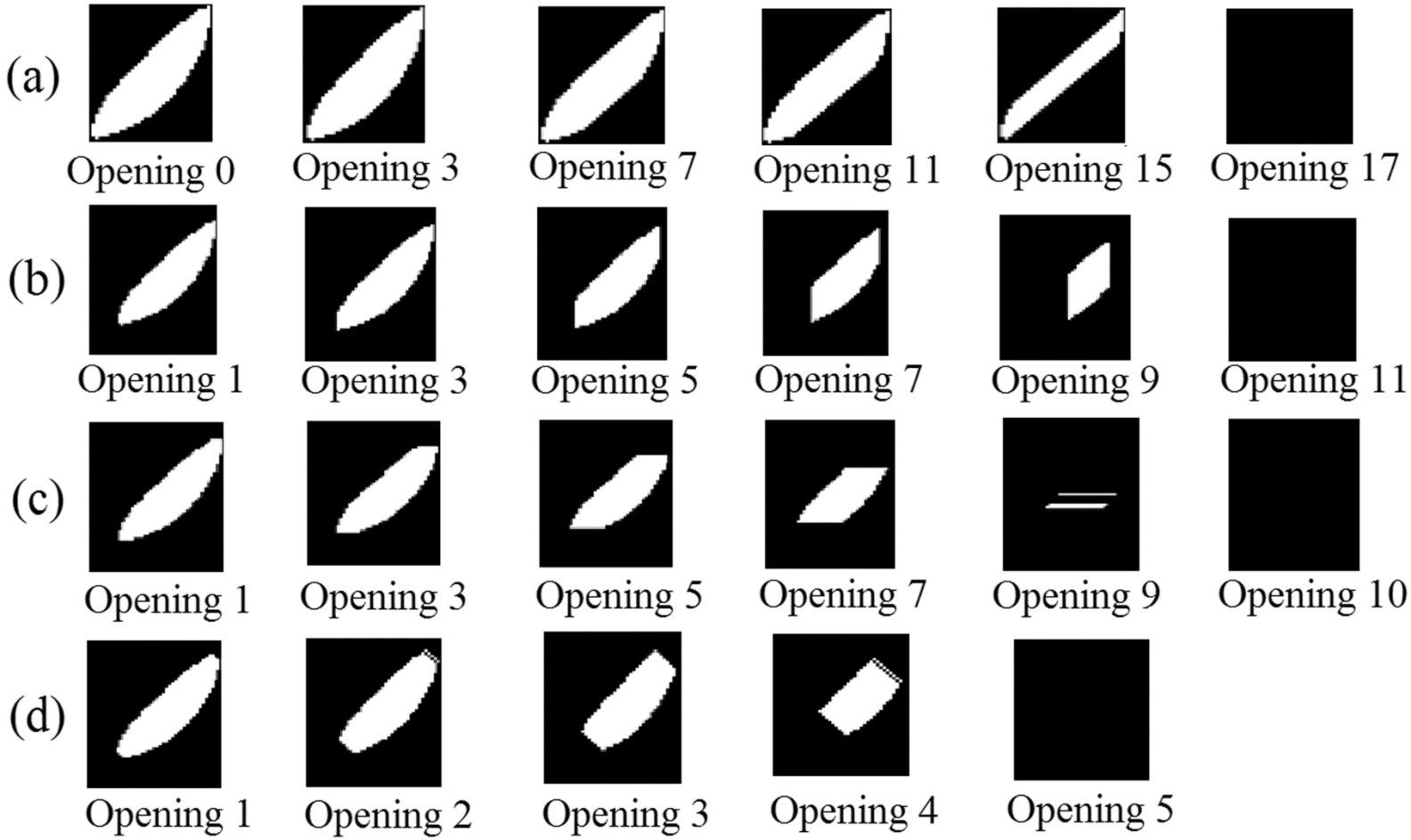
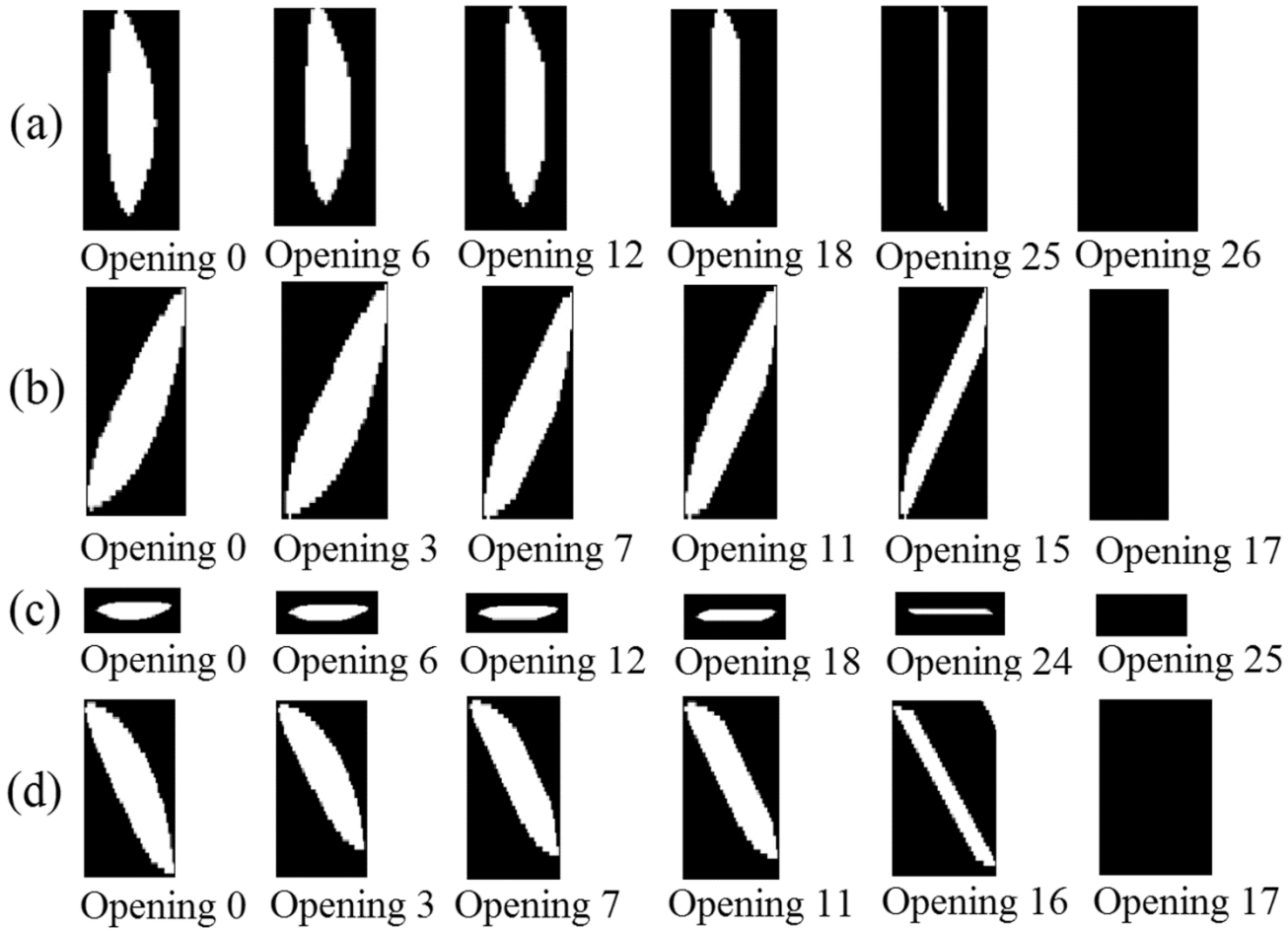
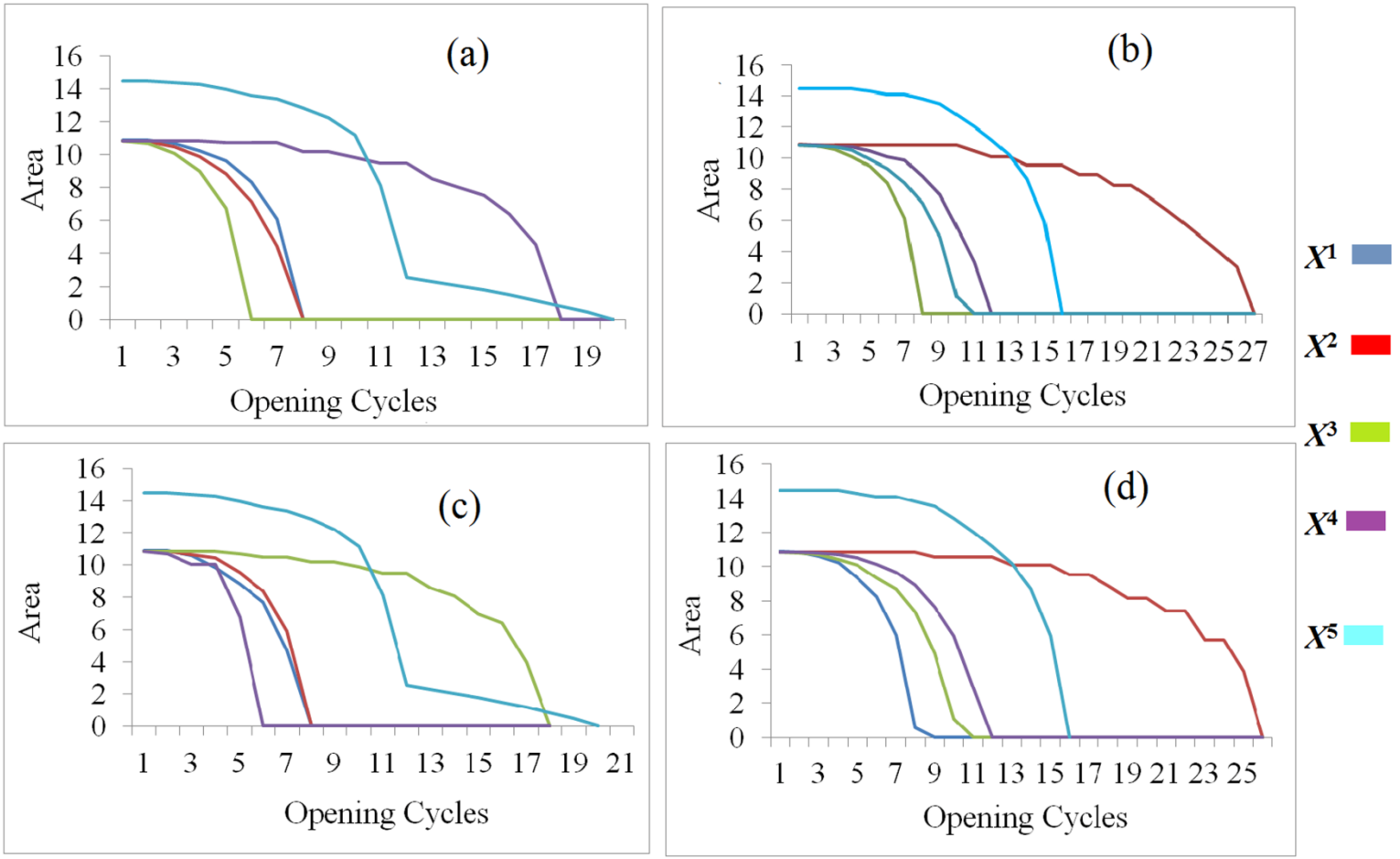
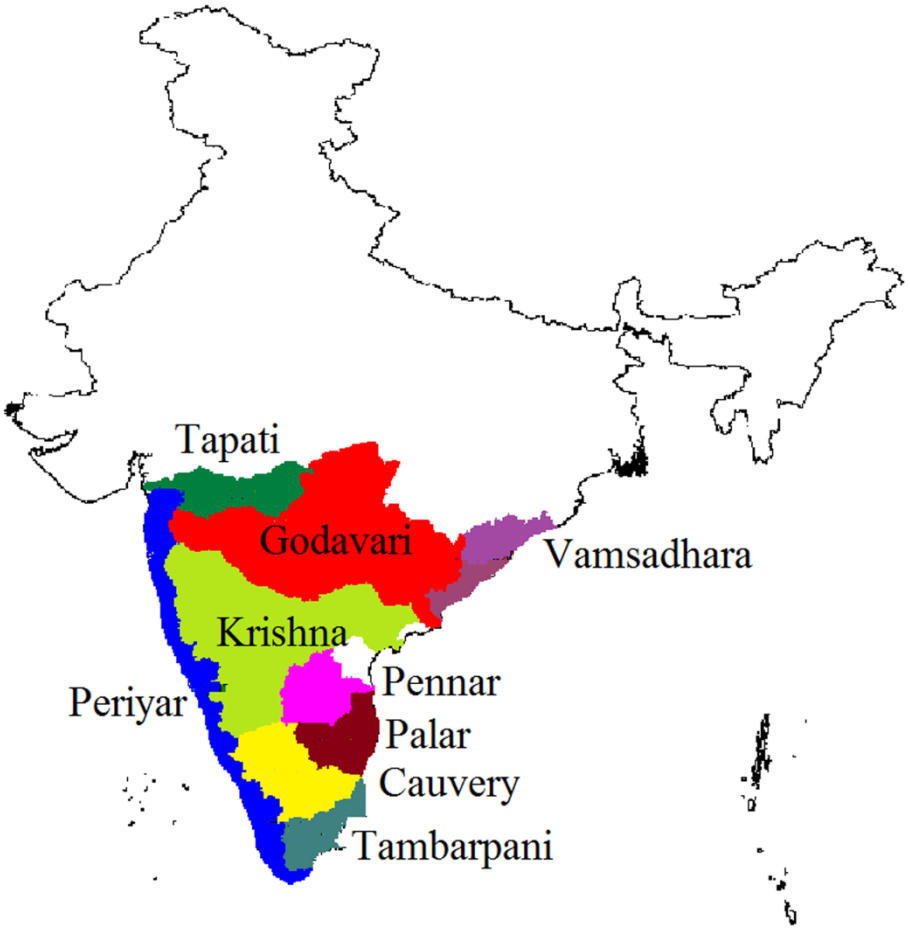
| 0 | 1 | 0 |
| 0 | 0 | 1 |

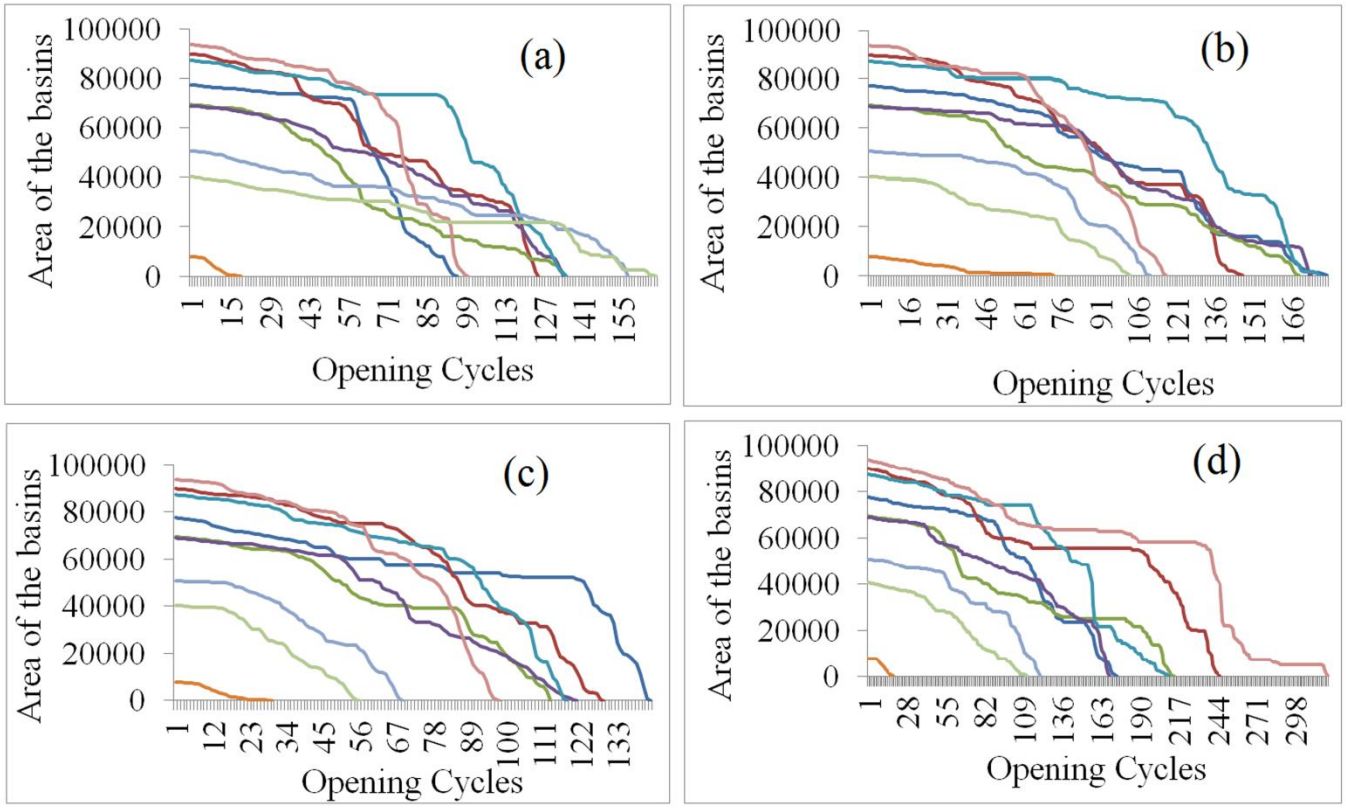

X2
X3
X5
X4
X1


(a)
Opening 0
Opening 3
Opening 7
Opening 11
Opening 15
Opening 17
(b)
Opening 1
Opening 3
Opening 5
Opening 7
Opening 9
Opening 11
(c)
Opening 1
Opening 3
Opening 5
Opening 7
Opening 9
Opening 10
(d)
Opening 1
Opening 2
Opening 3
Opening 4
Opening 5


(a)
Opening 0
Opening 6
Opening 12
Opening 18
Opening 25
Opening 26
(b)
Opening 0
Opening 3
Opening 7
Opening 11
Opening 15
Opening 17
(c)
Opening 0
Opening 6
Opening 12
Opening 18
Opening 24
Opening 25
(d)
Opening 0
Opening 3
Opening 7
Opening 11
Opening 16
Opening 17

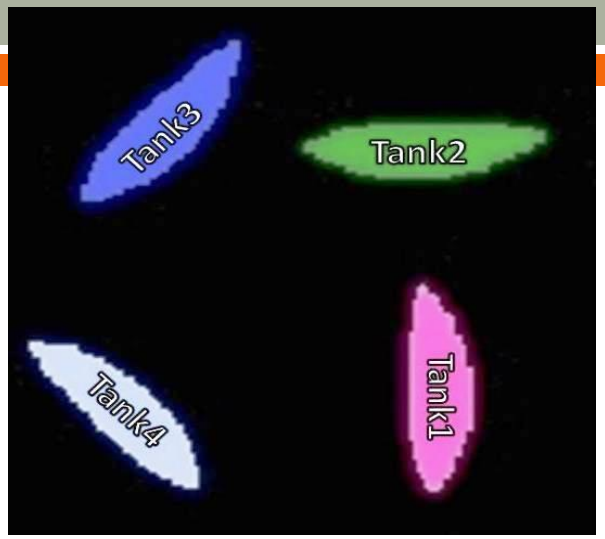

(a)
(b)
(c)
(d)
Area
Opening Cycles
X1
X2
X3
X4
X5


Tapati
Godavari
Vamsadhara
Krishna
Pennar
Periyar
Palar
Cauvery
Tambarpani


(a)
(b)
(c)
(d)
Area of the basins
Opening Cycles
100000
80000
60000
40000
20000
0
Cauvery
Godavari
Krishna
Palar
Pennar
Periyar
Tambarpani
Tapati
Vamsadhara

# Proposed Model

- **Input Image**: Different Symmetric Water bodies distributed in different directions.
- Structuring Elements (SEs) of different directions were considered.
- **Recursive Erosions** & **Recursive Dilations** were applied on each water body using each directional SE.
- **Morphological Entrophy** was computed via directional structuring elements.



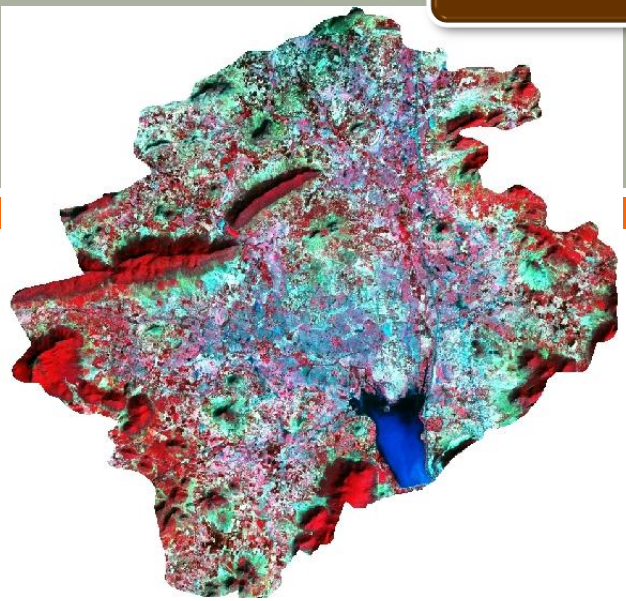

$$Pn\left(\frac{X_i}{B^i}\right) = \frac{A(X \circ n\, B^i) - A(X \circ (n+1)\, B^i)}{A(X)}$$

$$0 \le P_n(X_i/B^i) \le 1; \sum_{n=0}^{N} P_n\left(\frac{X_i}{B^i}\right) = 1$$

$$\left(\begin{matrix}Morphological\ Entrophy \\ of\ X_{i\ w.r.t}\ B^i\end{matrix}\right) = -\sum_{n=0}^{N} P_n\left(\frac{X_i}{B^i}\right) \log P_n\left(\frac{X_i}{B^i}\right)$$

- Directional behaviour of each water body with respect to each SE was observed.
- Granulometric Indices were recorded for each iteration.

# OVERALL METHODOLOGY



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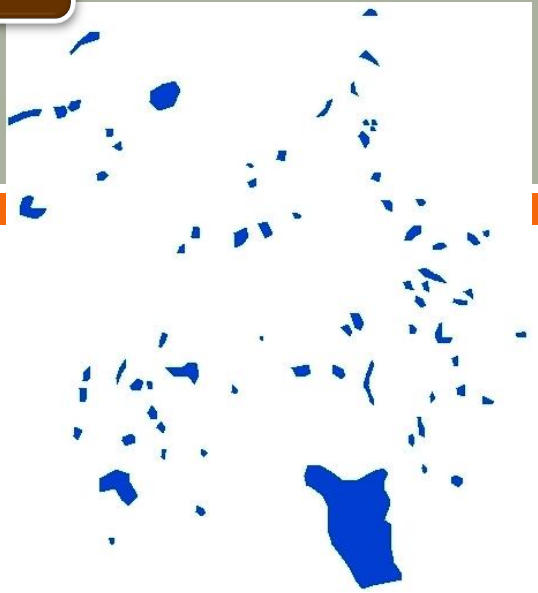

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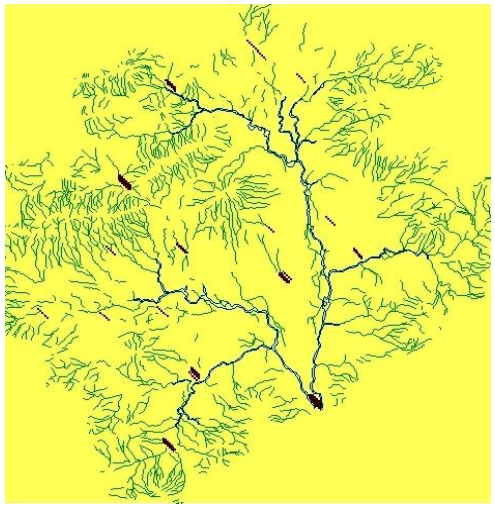
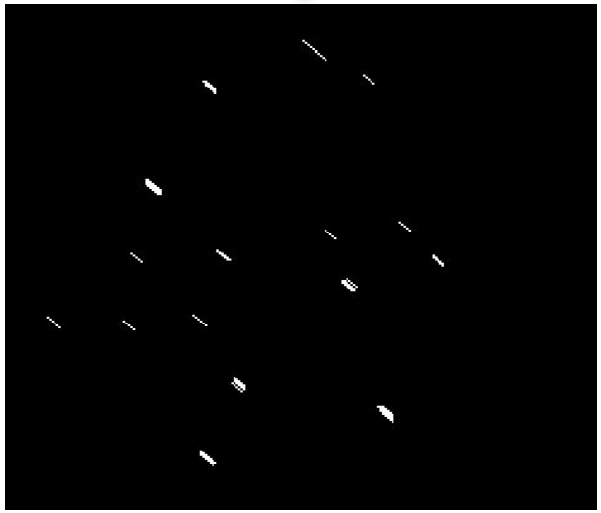


## Recursive Openings



## 1-D Structuring Elements

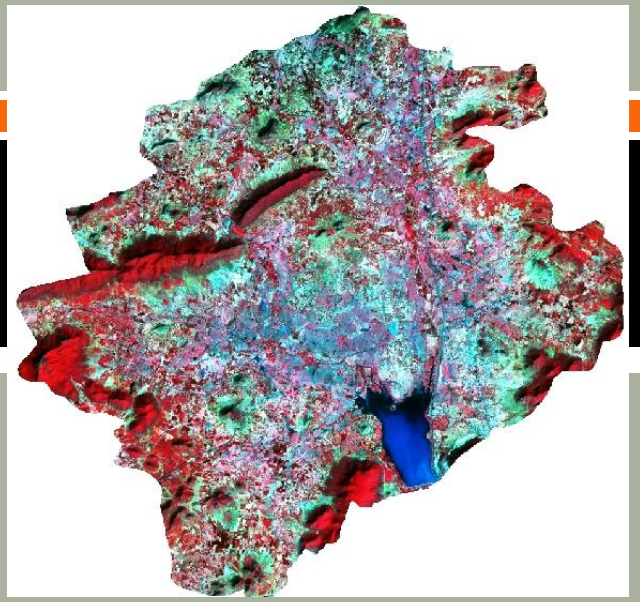
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 0 | 0 | 0 |

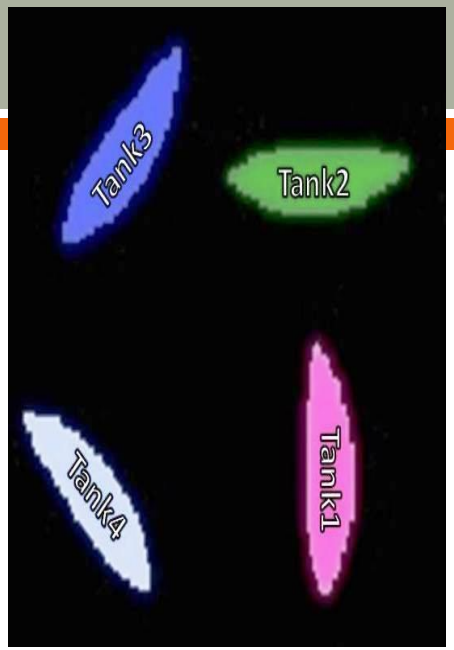
| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

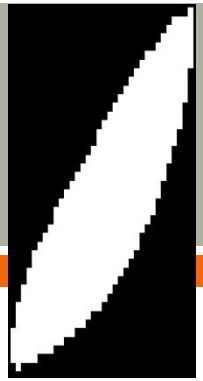
| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

# REMOTE SENSING DATA

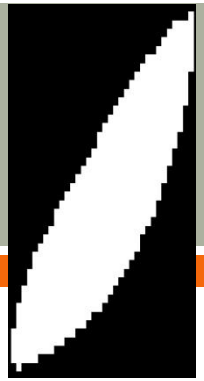





| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |



Opening 0



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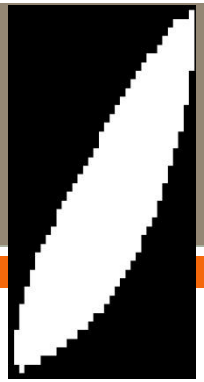

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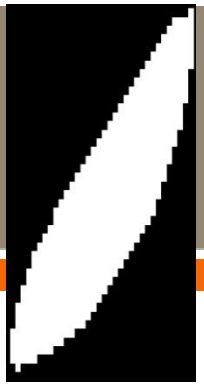
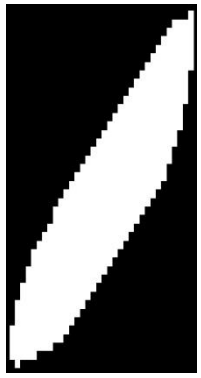


Opening 5

Opening 7



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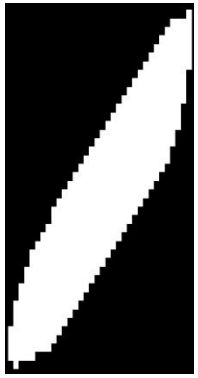

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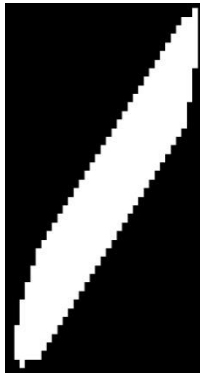

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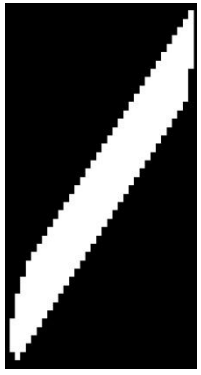

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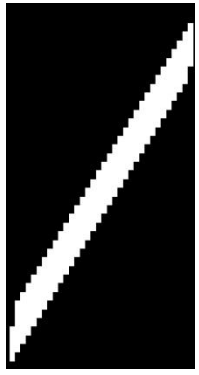
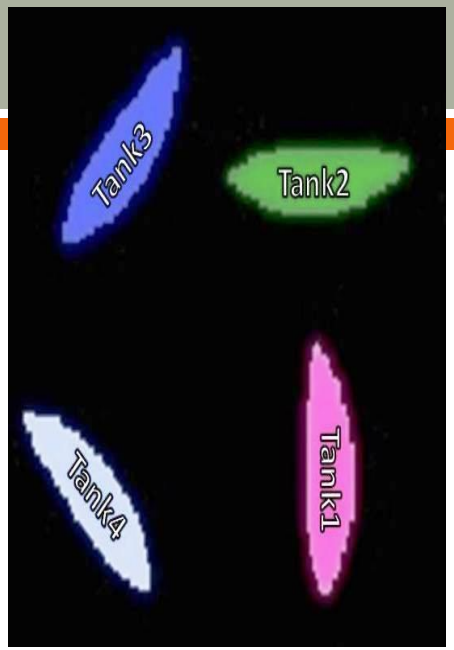


Opening 16

Opening 17

# Input Image:




| | | |
|---|---|---|
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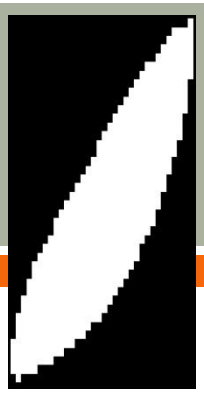
| 1 | 1 | 1 |
| 0 | 0 | 0 |



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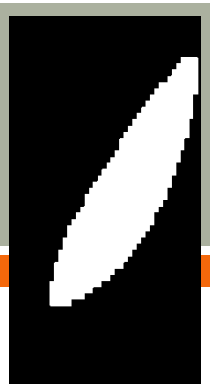

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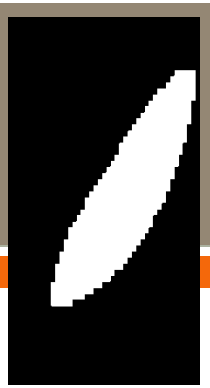

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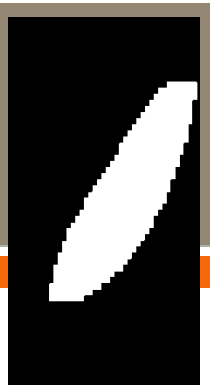

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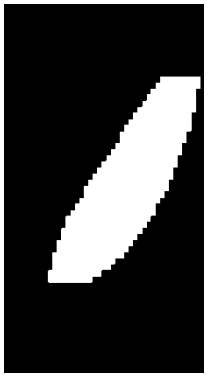

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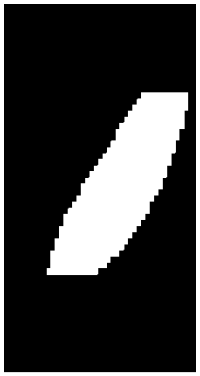
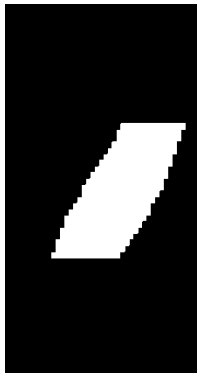


Opening 5

Opening 6



Opening 7

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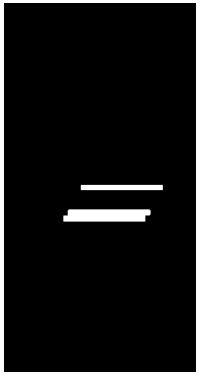

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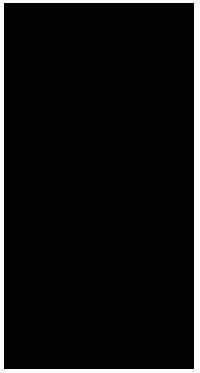

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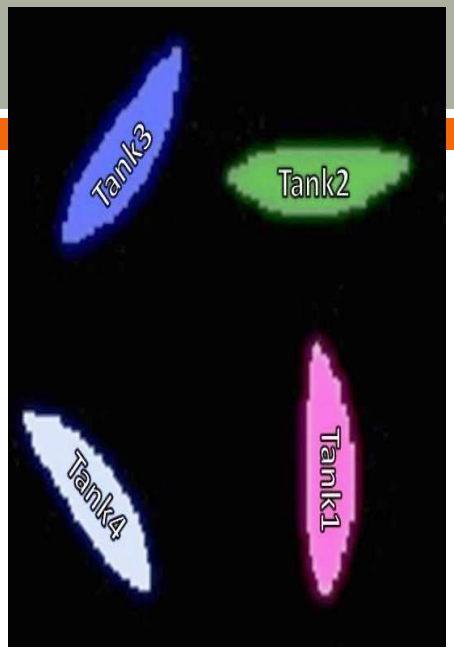
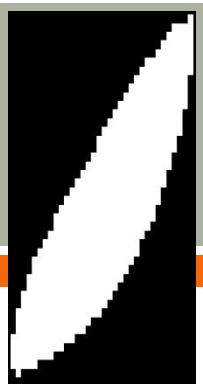



| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |



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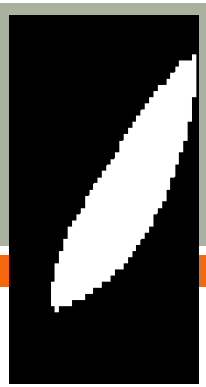

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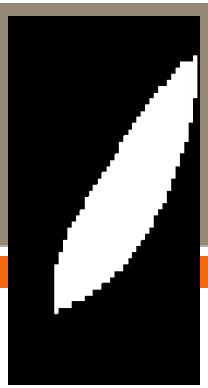

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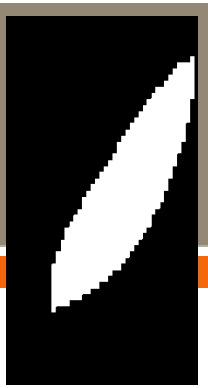

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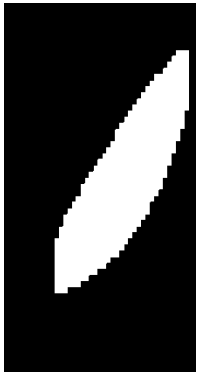

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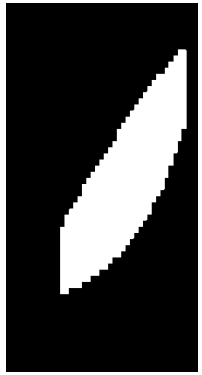

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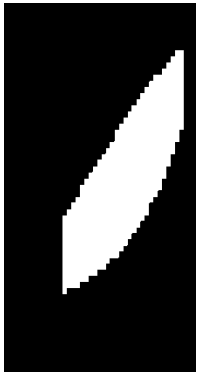

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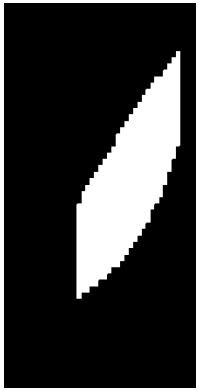

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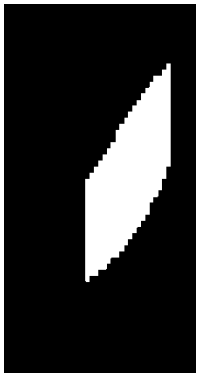
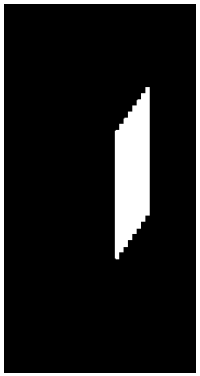


Opening 8

Opening 9



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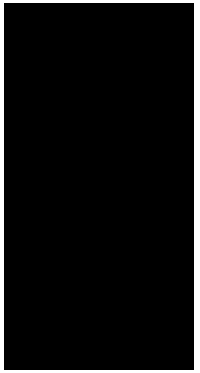
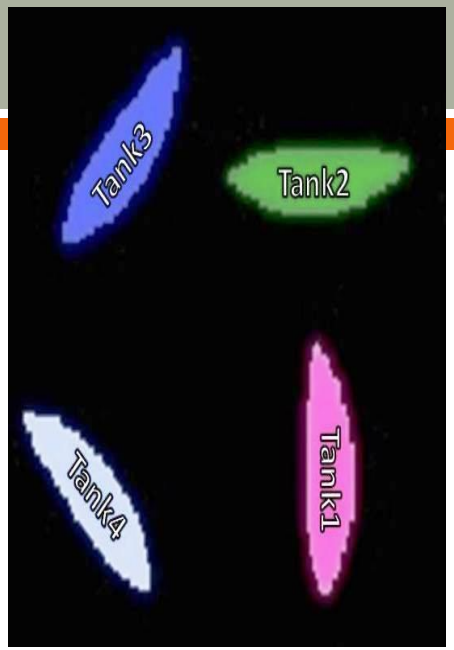
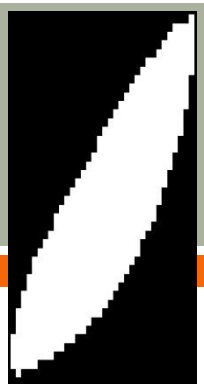
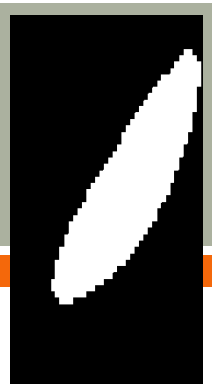


Opening 11

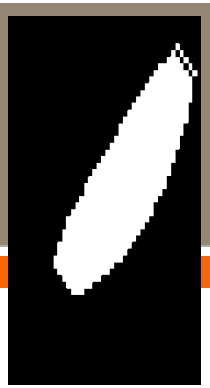


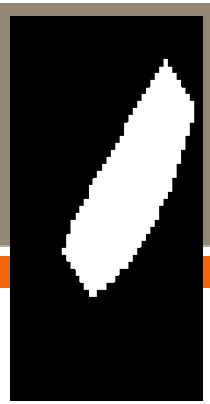
Opening 0



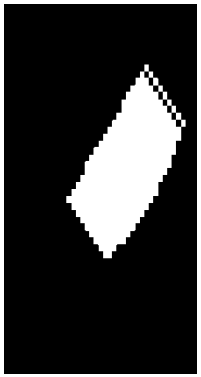
Opening 1



Opening 2



Opening 3



Opening 4

Opening 5

| | | |
|---|---|---|
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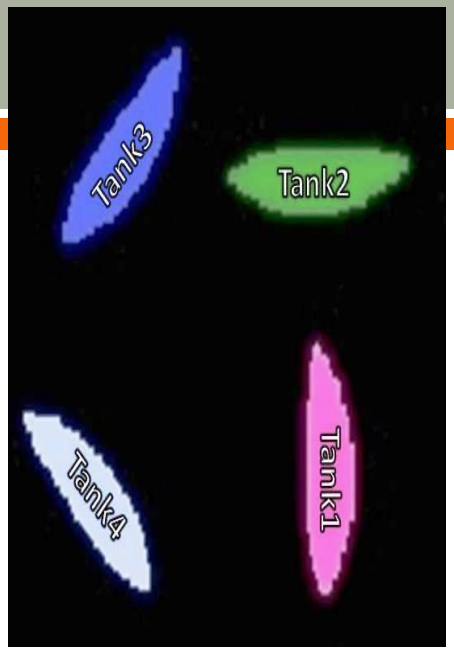
| 0 | 1 | 0 |
| 0 | 0 | 1 |



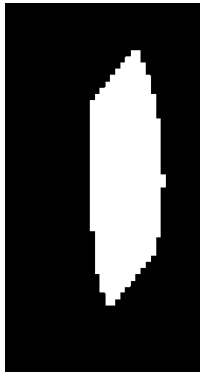

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

Opening 0

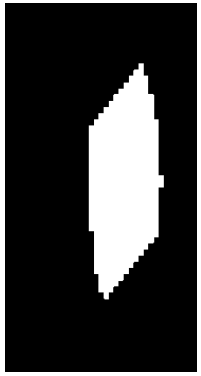
Opening 1

Opening 2

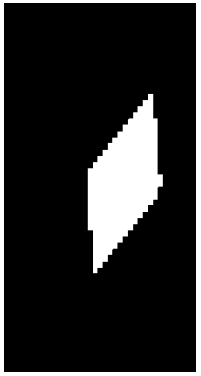
Opening 3



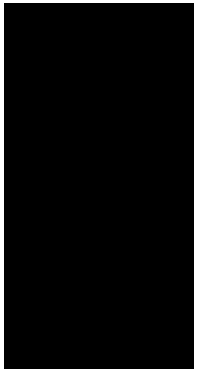
Opening 4



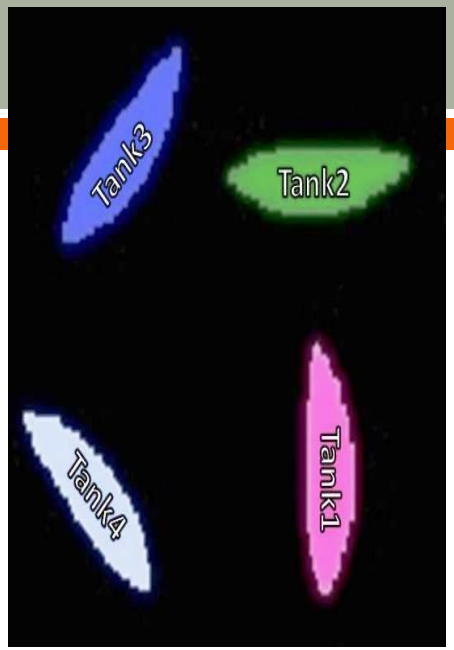
Opening 5



Opening 6



Opening 7




| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

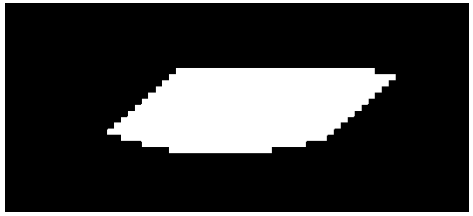
**Opening 0**

**Opening 1**

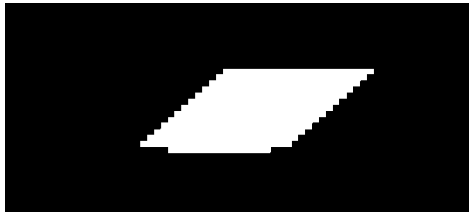
**Opening 2**

**Opening 3**

**Opening 4**

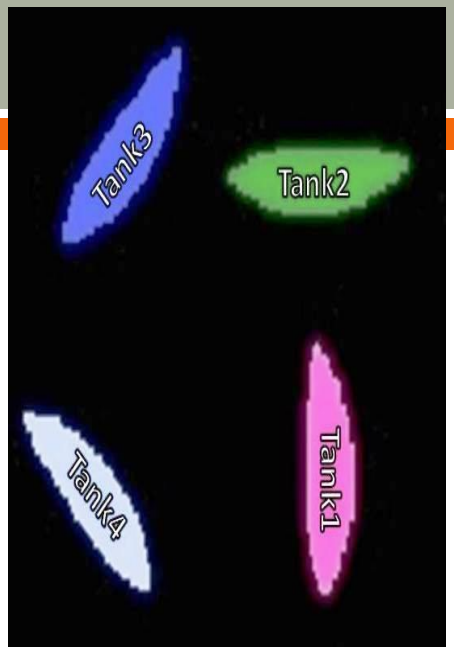


**Opening 5**

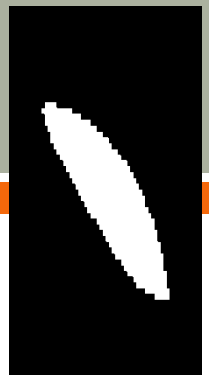


**Opening 6**

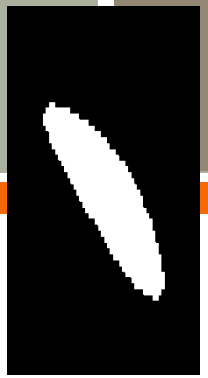
**Opening 7**



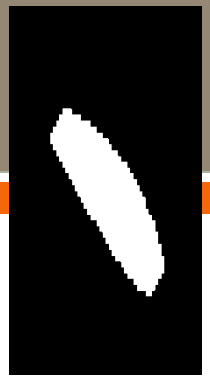

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |



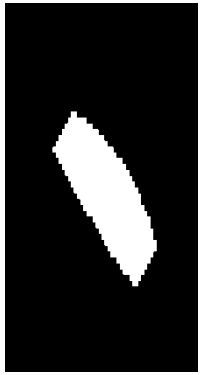
**Opening 0**



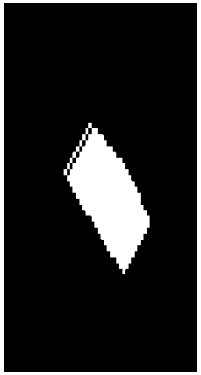
**Opening 1**



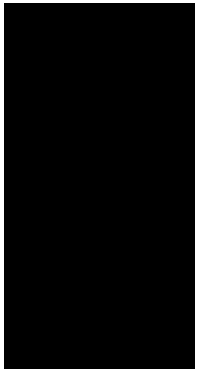
**Opening 2**



**Opening 3**



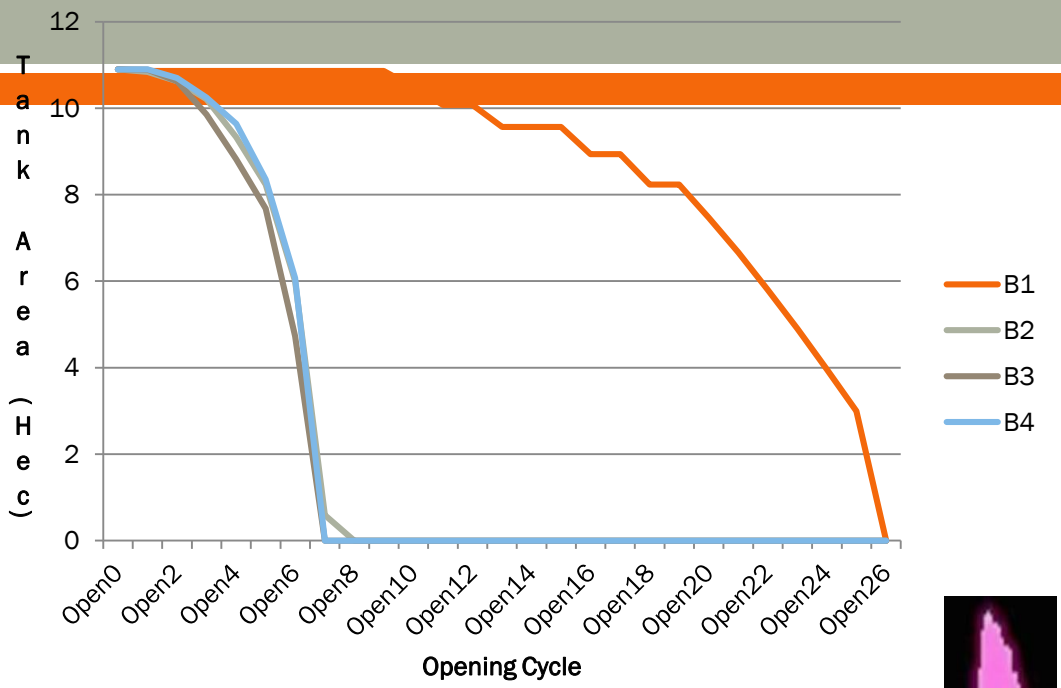
**Opening 4**



**Opening 5**

# Area of the Water bodies w.r.t ROT

| Opening Cycle | $B^1$ | $B^2$ | $B^3$ | $B^4$ |
|---|---|---|---|---|
| Open0 | 10.896 | 10.896 | 10.896 | 10.896 |
| Open1 | 10.8584 | 10.8388 | 10.891 | 10.8976 |
| Open2 | 10.8584 | 10.6232 | 10.623 | 10.7016 |
| Open3 | 10.8584 | 10.2116 | 9.8588 | 10.2508 |
| Open4 | 10.8584 | 9.3296 | 8.82 | 9.6432 |
| Open5 | 10.8584 | 8.232 | 7.6832 | 8.3496 |
| Open6 | 10.8584 | 5.9976 | 4.7236 | 6.076 |
| Open7 | 10.8584 | 0.588 | 0 | 0 |
| Open8 | 10.8584 | 0 | 0 | 0 |
| Open9 | 10.8584 | 0 | 0 | 0 |
| Open10 | 10.486 | 0 | 0 | 0 |
| Open11 | 10.0744 | 0 | 0 | 0 |
| Open12 | 10.0744 | 0 | 0 | 0 |
| Open13 | 9.5648 | 0 | 0 | 0 |
| Open14 | 9.5648 | 0 | 0 | 0 |
| Open15 | 9.5648 | 0 | 0 | 0 |
| Open16 | 8.9376 | 0 | 0 | 0 |
| Open17 | 8.9376 | 0 | 0 | 0 |
| Open18 | 8.232 | 0 | 0 | 0 |
| Open19 | 8.232 | 0 | 0 | 0 |
| Open20 | 7.4676 | 0 | 0 | 0 |
| Open21 | 6.664 | 0 | 0 | 0 |
| Open22 | 5.8016 | 0 | 0 | 0 |
| Open23 | 4.9 | 0 | 0 | 0 |
| Open24 | 3.9592 | 0 | 0 | 0 |
| Open25 | 2.9988 | 0 | 0 | 0 |
| Open26 | 0 | 0 | 0 | 0 |




## Structuring Element:

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

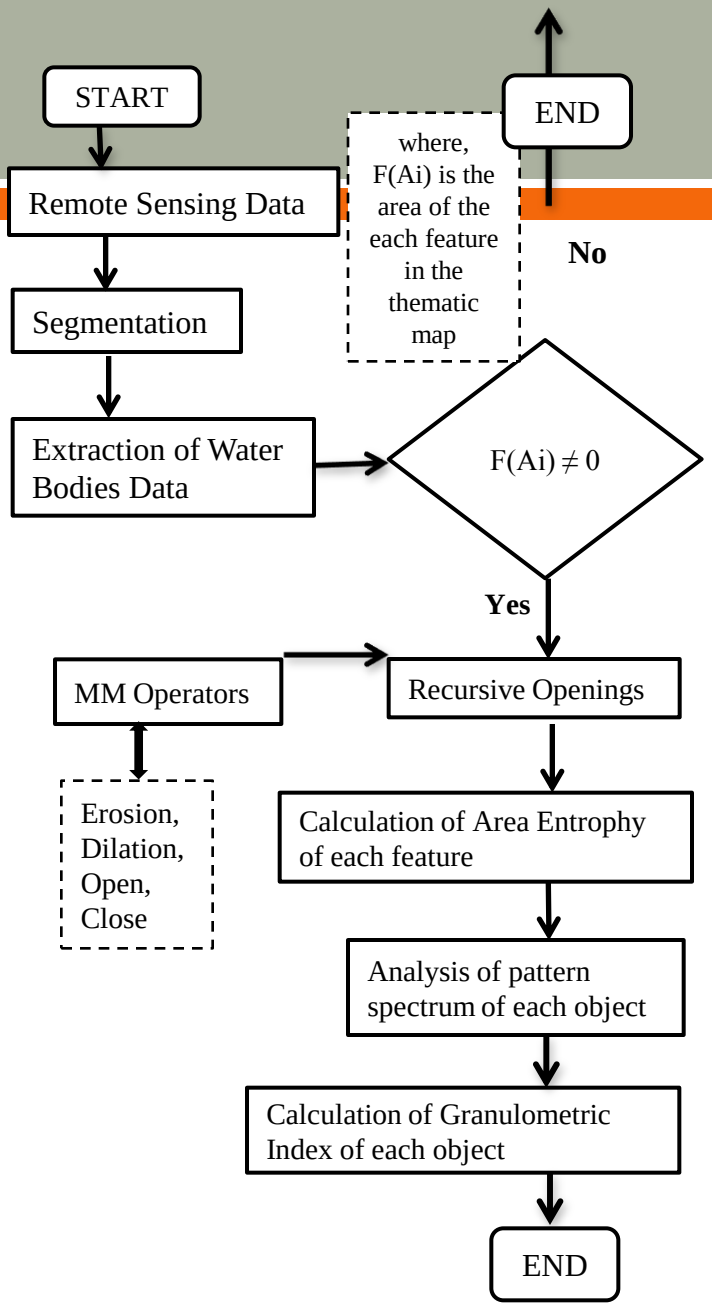
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

# GRANULOMETRIC INDEX

| SE | Tank-1 | Tank-2 | Tank-3 | Tank-4 |
|---|---|---|---|---|
| 0 1 0<br>0 1 0<br>0 1 0 | 1.898 | 0.678 | 0.891 | 0.789 |
| 0 0 0<br>1 1 1<br>0 0 0 | 0.7325 | 1.789 | 0.8225 | 0.7635 |
| 0 0 1<br>0 1 0<br>1 0 0 | 0.6329 | 0.5504 | 0.8220 | 0.3897 |
| 1 0 0<br>0 1 0<br>0 0 1 | 0.6321 | 0.578 | 0.389 | 0.8320 |





START
Remote Sensing Data
Segmentation
Extraction of Water Bodies Data
F(Ai) ≠ 0
No
END
where, F(Ai) is the area of the each feature in the thematic map
Yes
MM Operators
Recursive Openings
Erosion, Dilation, Open, Close
Calculation of Area Entrophy of each feature
Analysis of pattern spectrum of each object
Calculation of Granulometric Index of each object
END