MATHEMATICAL MORPHOLOGY: MORPHOLOGICAL SHAPE DECOMPOSITION

B.S. DAYA SAGAR

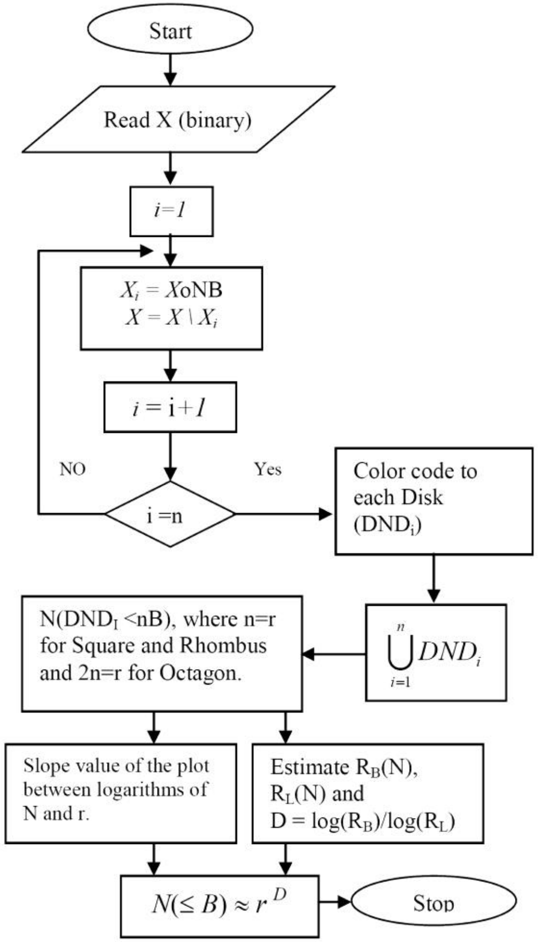
http://www.isibang.ac.in/~bsdsagar

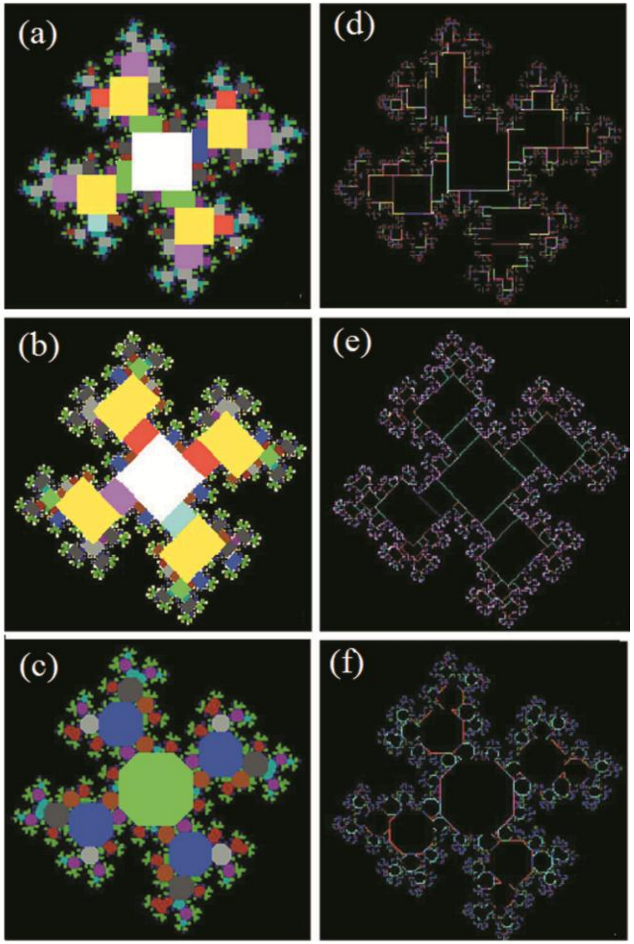
Systems Science and Informatics Unit (SSIU)

Indian Statistical Institute-Bangalore Centre

INDIAN STATISTICAL INSTITUTE

Bangalore Centre


Start
Read X (binary)
i=1
$X_i = X o NB$
$X = X \setminus X_i$
i = i+1
NO
Yes
i =n
Color code to each Disk (DND_i)
$\bigcup_{i=1}^{n} DND_i$
N(DND_I <nB), where n=r for Square and Rhombus and 2n=r for Octagon.
Slope value of the plot between logarithms of N and r.
Estimate R_B(N), R_L(N) and D = log(R_B)/log(R_L)
$N(\leq B) \approx r^D$
Stop


(a)
(d)
(b)
(e)
(c)
(f)

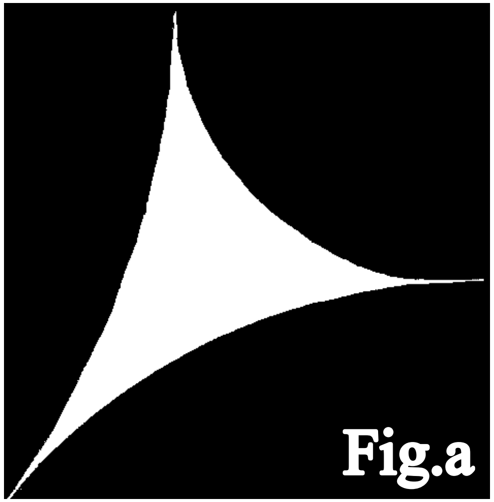


Fig.a

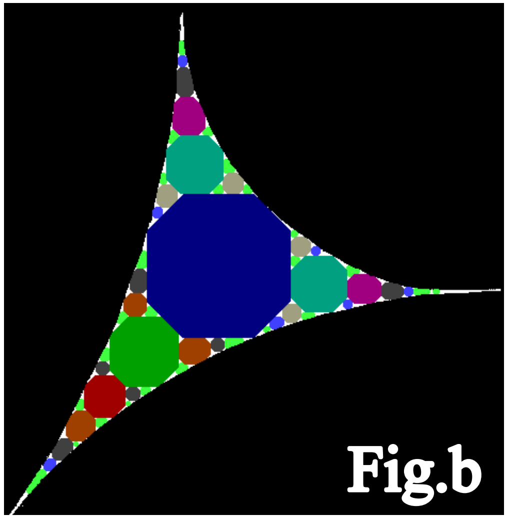
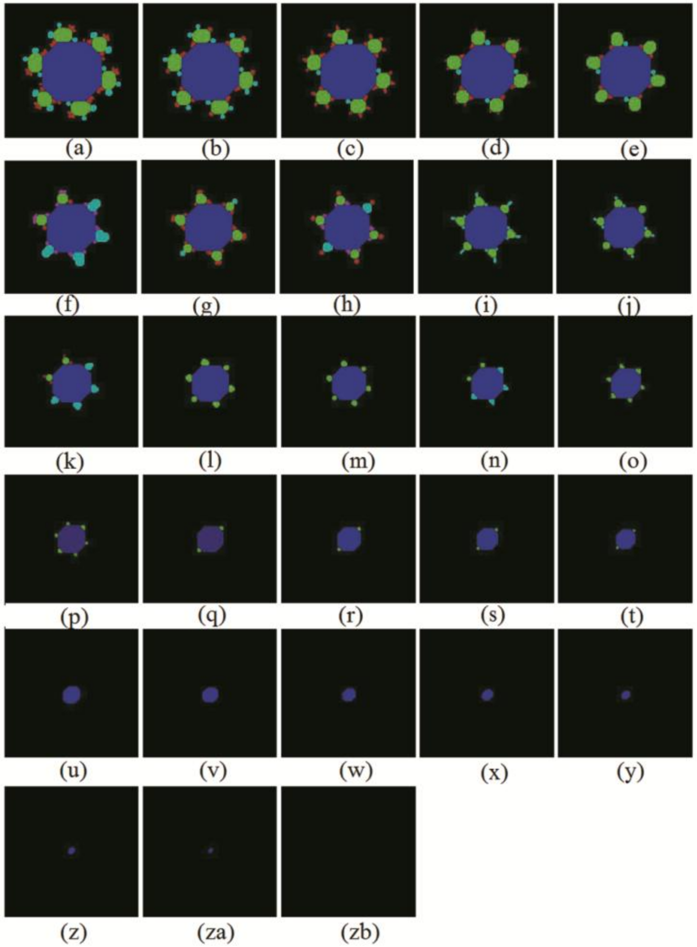


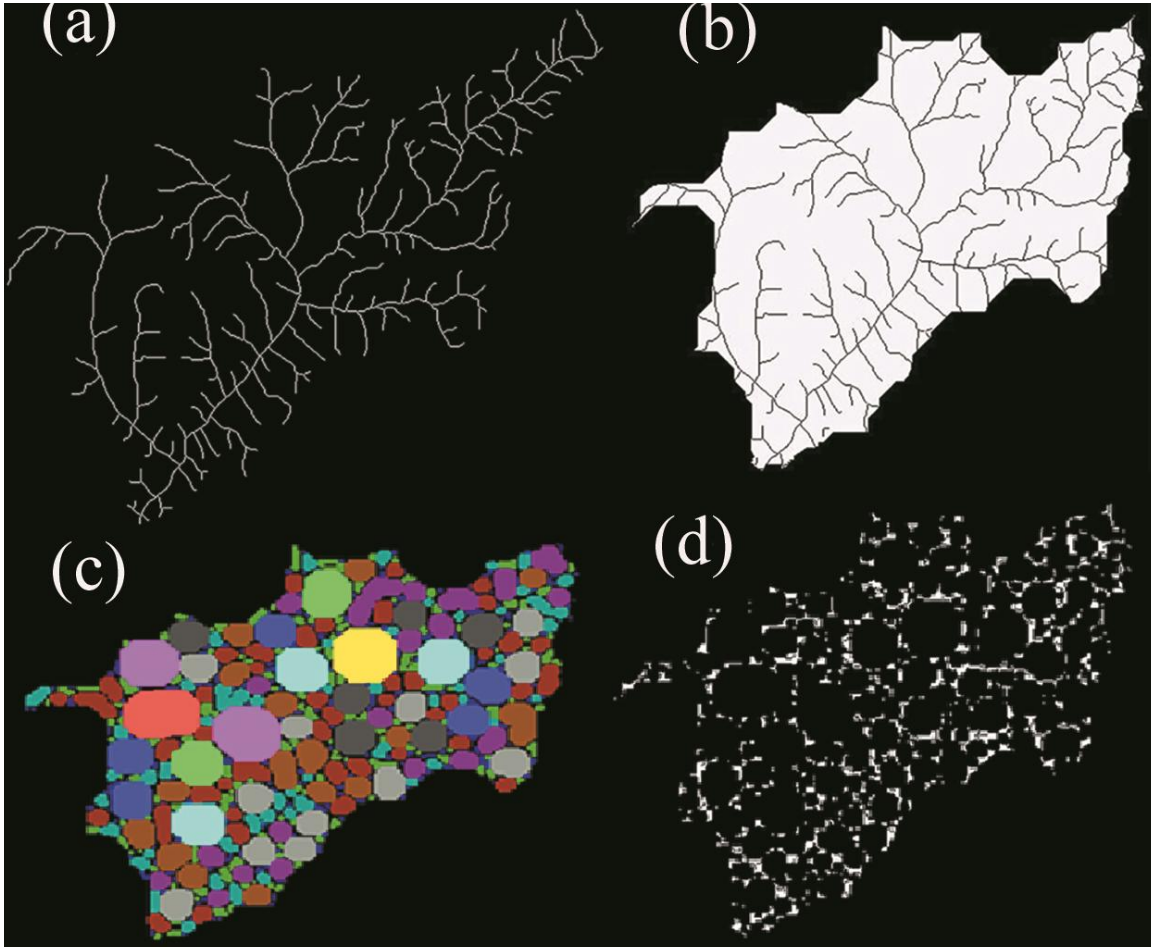
Fig.b

Power law relationship

- As per the previous fig. the slopes of the best-fit lines (α_N and α_A) for number-radius and area-radius relationships yield 2.37 and 1.34.
- These slope values of the best-fit lines provide shape dependent dimensions as $D_N = \alpha_N - 1$ and $D_A = \alpha_A$.
- As in previous Fig., D_N and D_A for non-network space yield 1.37 and 1.34.
- A Power-law relationship is shown in earlier Fig. with an exponent value 1.79 between the area and number of NODs observed with increasing radius of structuring template.

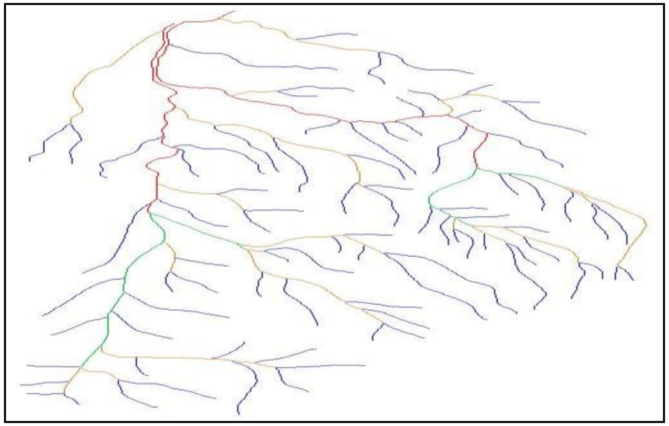
(a) Appollonian Space, and (b) after decomposition by means of octagon.


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)
(p)
(q)
(r)
(s)
(t)
(u)
(v)
(w)
(x)
(y)
(z)
(za)
(zb)

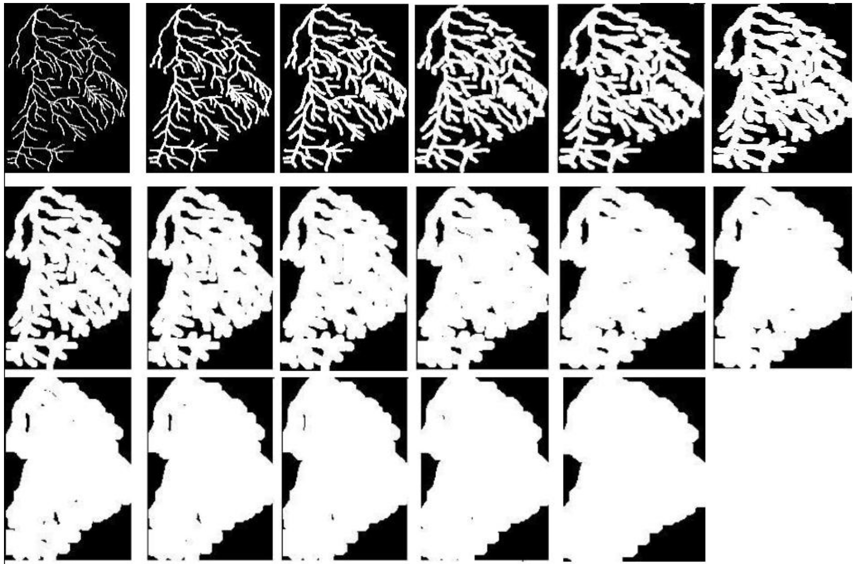

(a)
(b)
(c)
(d)

Algorithm Implementation:

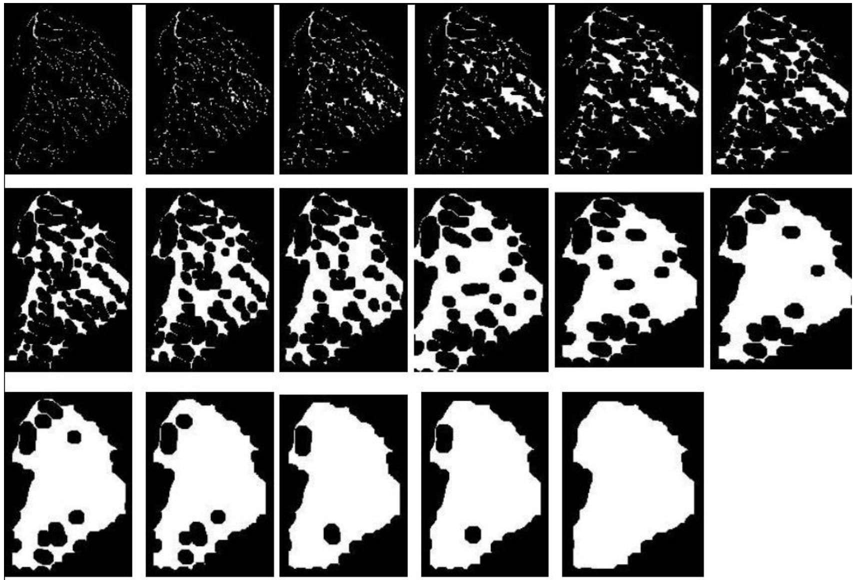
Step 1: **Channel network of sub basin 1**



Step 2: **Close-Hull Generation**

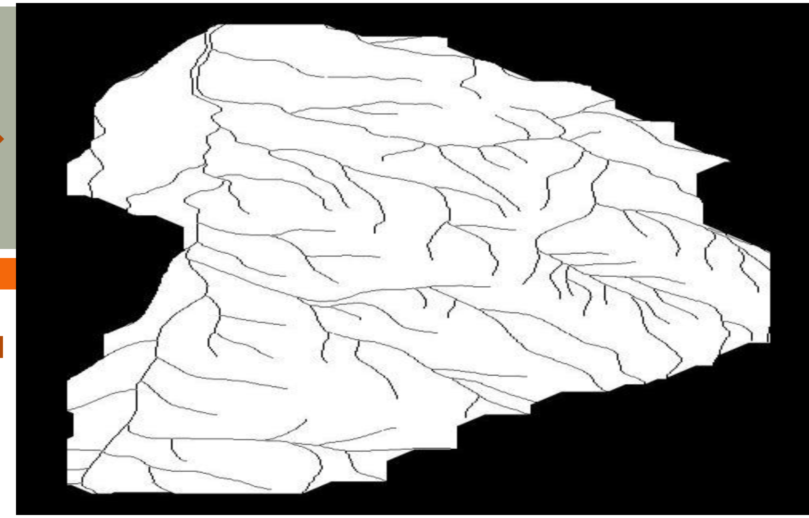


Iterative dilation of channel network of basin 1

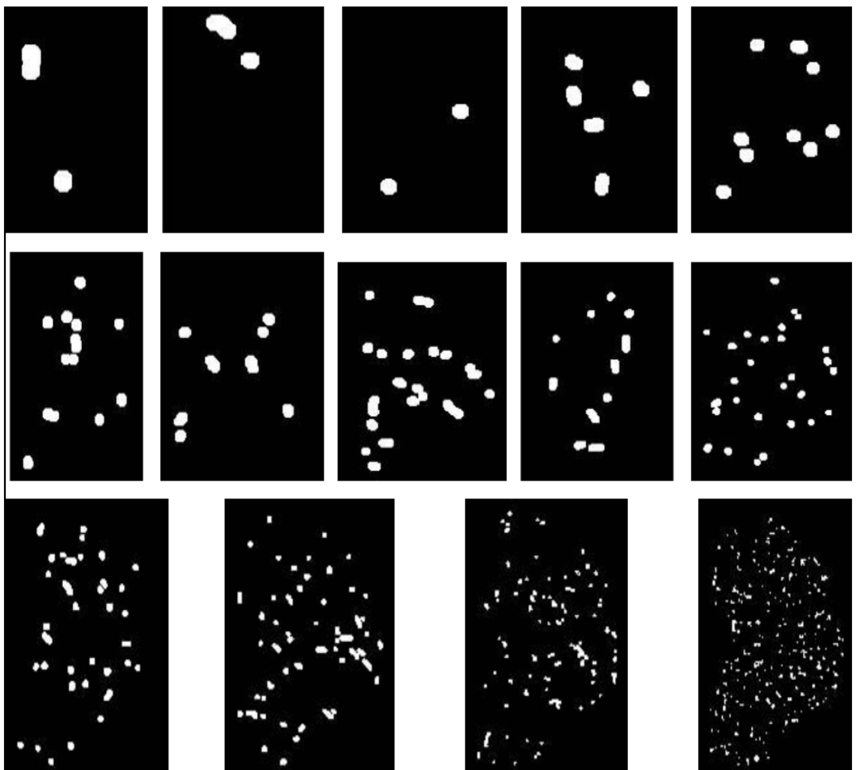


Iterative erosion applied to previous Fig

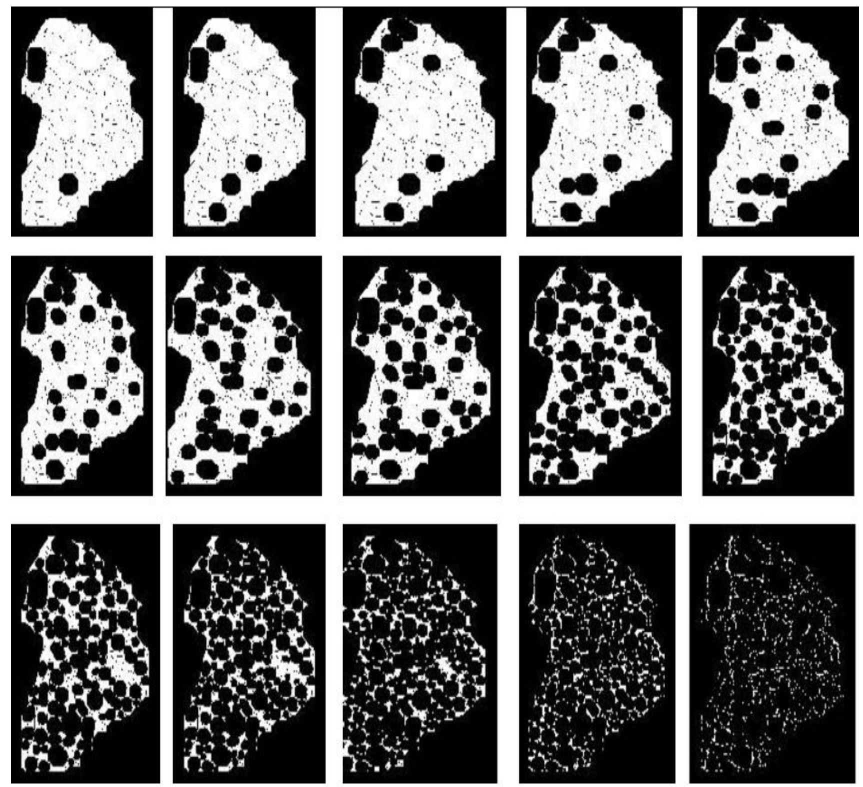
Step 3: Non-network space of basin 1



Iterative erosion applied to step-3 Fig.

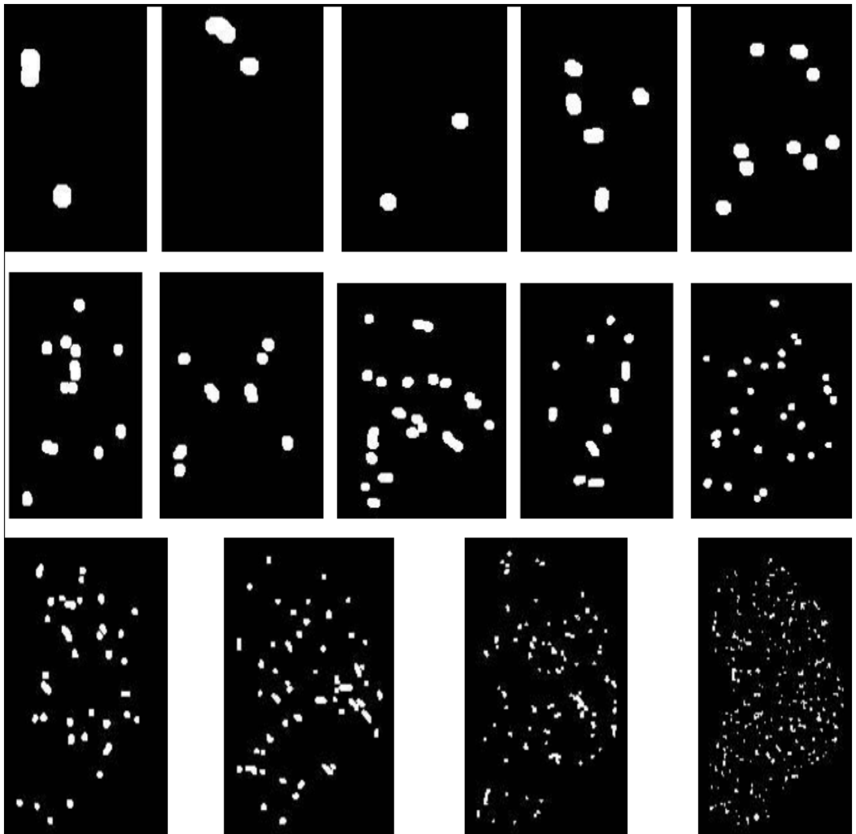


Iterative erosion applied to previous Fig.

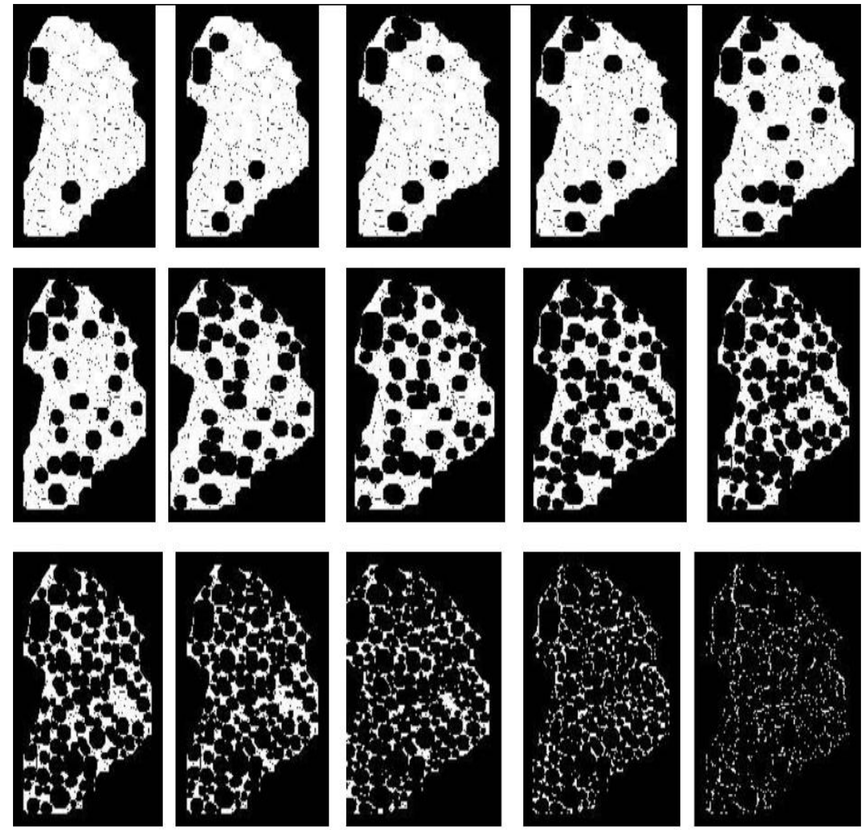


Iterative dilation applied to previous Fig.

Step 4: Non-Network Space Decomposition



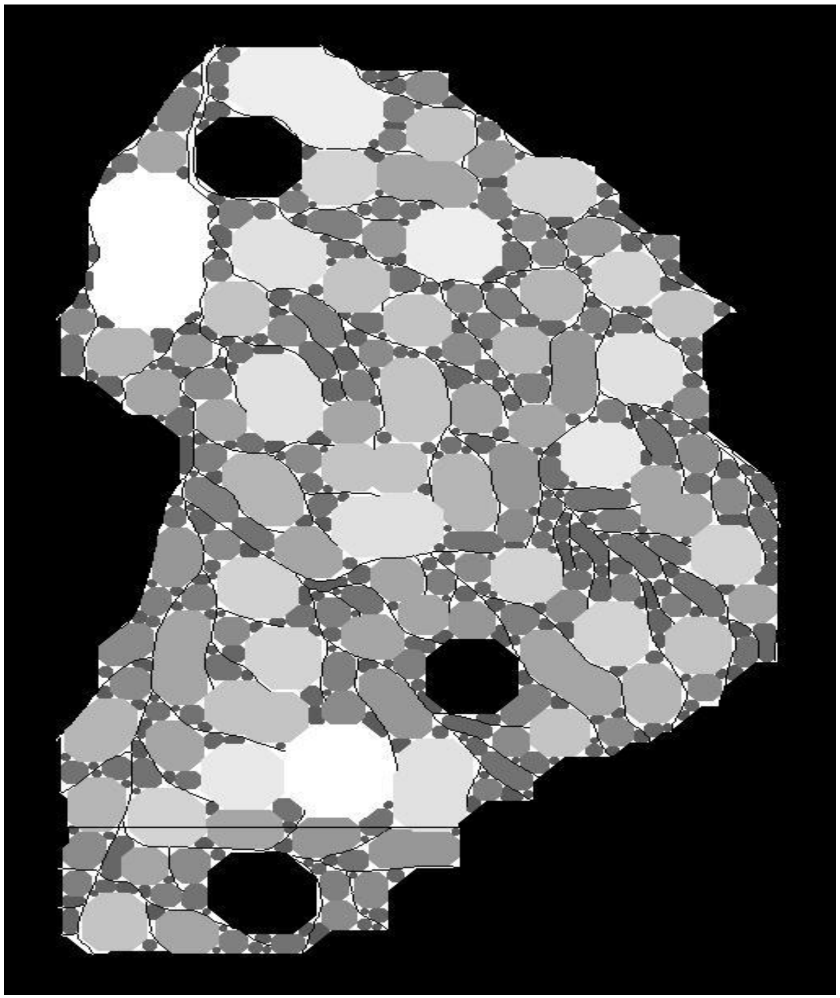
Iterative erosion applied to previous Fig.

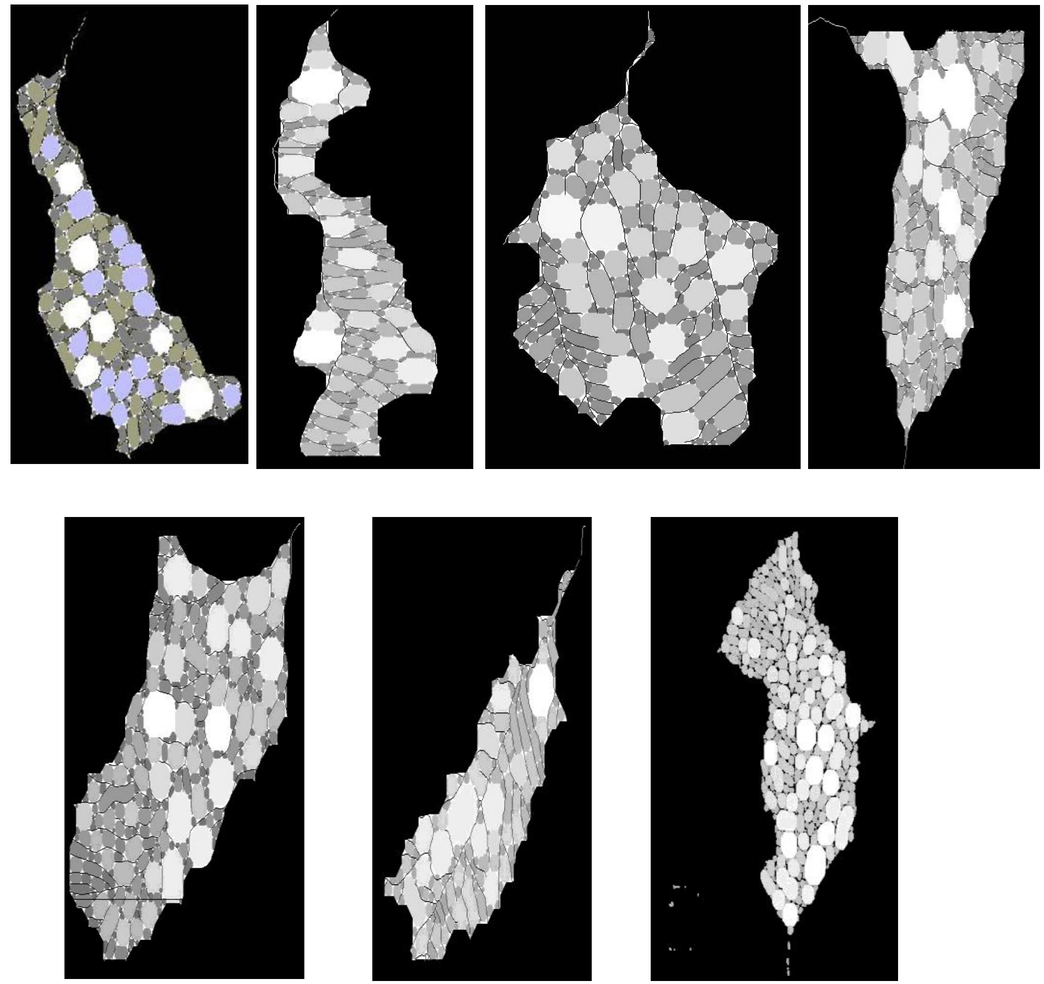


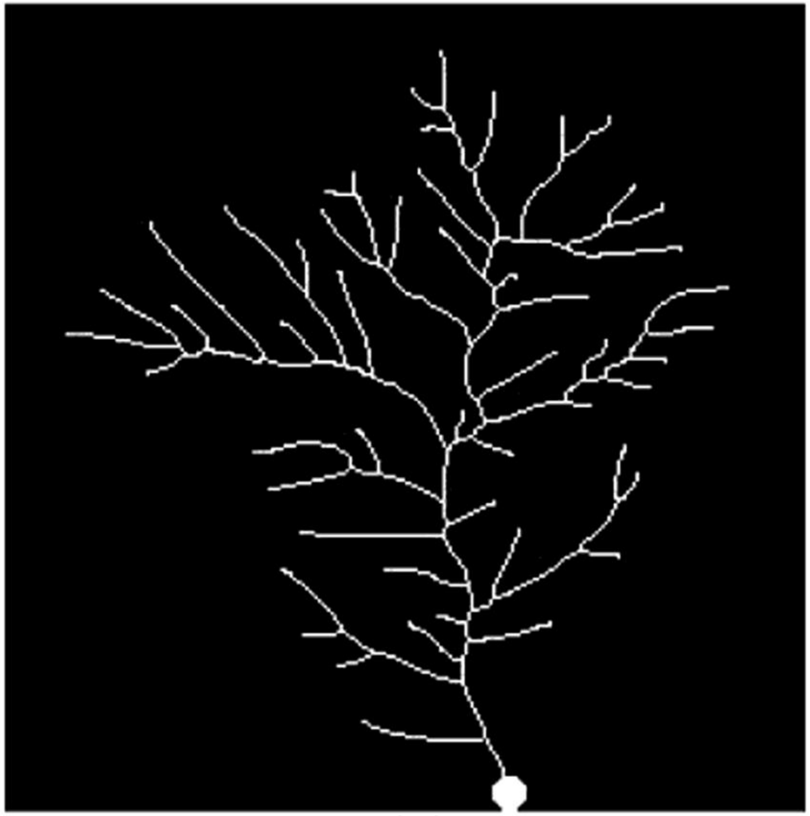
Iterative dilation applied to previous Fig.

Decomposition of Non-network space in to non-overlapping disks of octagon shape of several sizes for basin 1

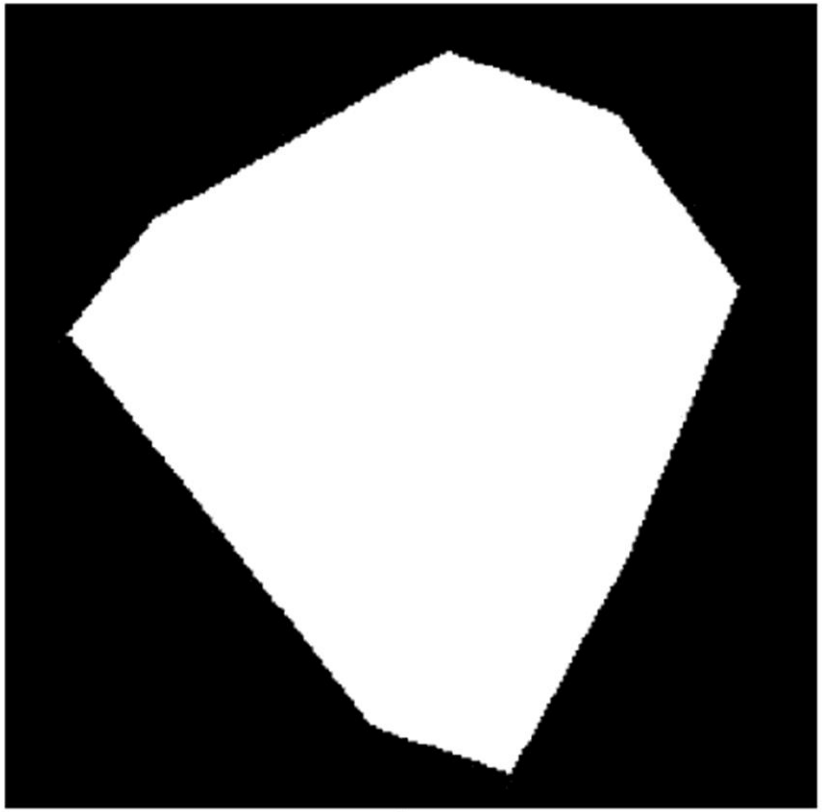
Non-Network Spaces Packed with Non-Overlapping Disks of basins 2 to 8







(a)



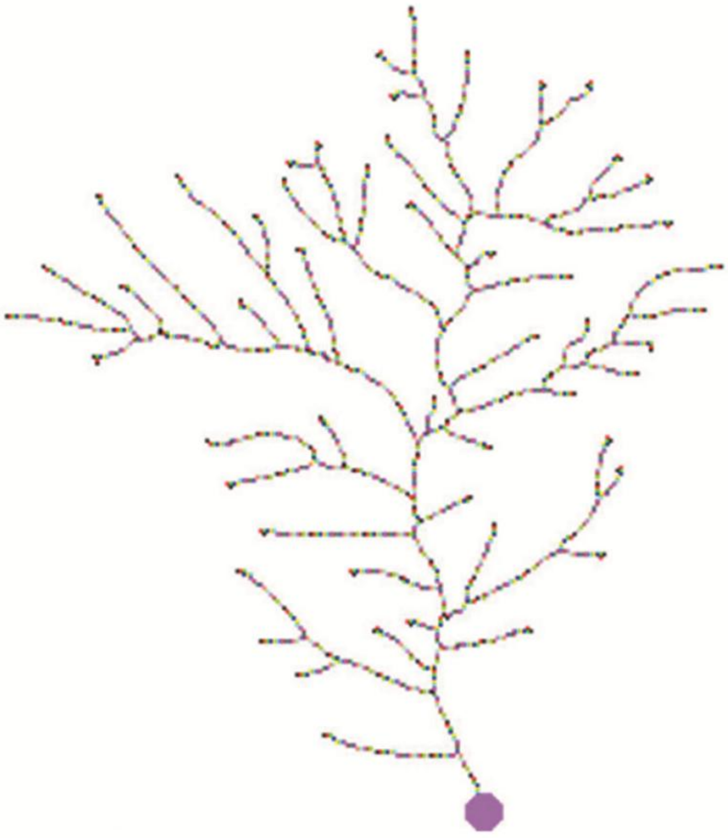
(b)

$$B_1^1 = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \quad B_1^2 = \begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \quad B_1^3 = \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \quad B_1^4 = \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}$$

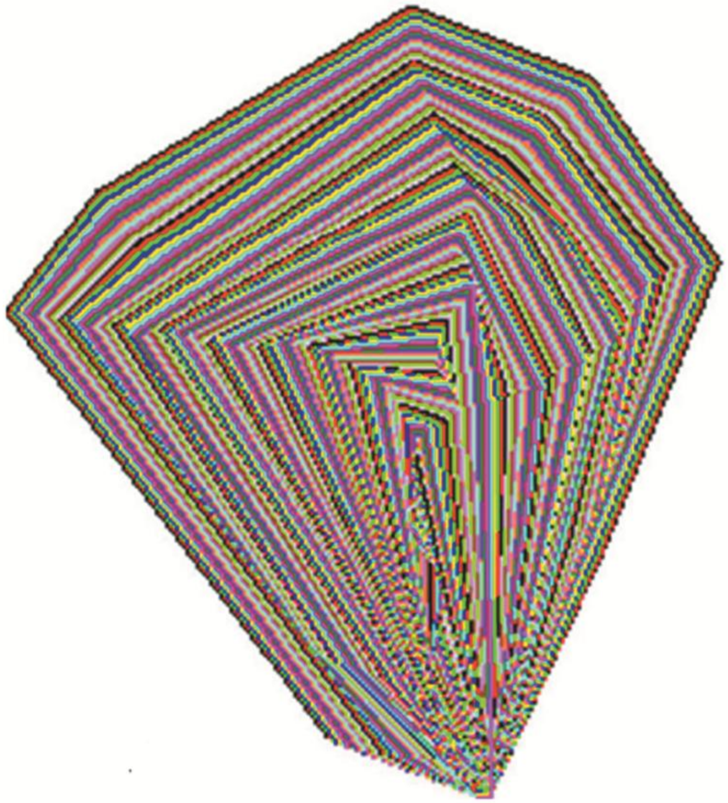
$$B_1^5 = \begin{matrix} X & 1 & X \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \quad B_1^6 = \begin{matrix} 0 & 0 & X \\ 0 & 1 & 1 \\ 0 & 0 & X \end{matrix} \quad B_1^7 = \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ X & 1 & X \end{matrix} \quad B_1^8 = \begin{matrix} X & 0 & 0 \\ 1 & 1 & 0 \\ X & 0 & 0 \end{matrix}$$

$$B_2^1 = \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix} \quad B_2^2 = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix} \quad B_2^3 = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix} \quad B_2^4 = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{matrix}$$

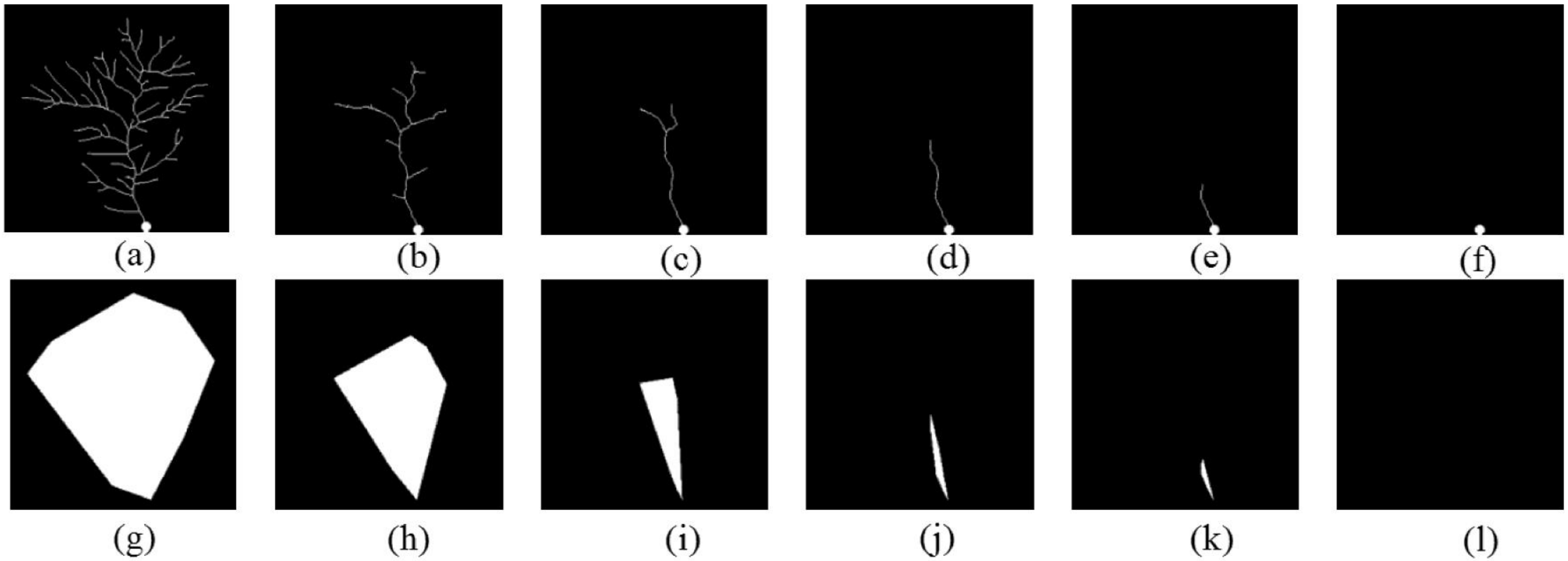
$$B_2^5 = \begin{matrix} X & 0 & X \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix} \quad B_2^6 = \begin{matrix} 1 & 1 & X \\ 1 & 0 & 0 \\ 1 & 1 & X \end{matrix} \quad B_2^7 = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ X & 0 & X \end{matrix} \quad B_2^8 = \begin{matrix} X & 1 & 1 \\ 0 & 0 & 1 \\ X & 1 & 1 \end{matrix}$$



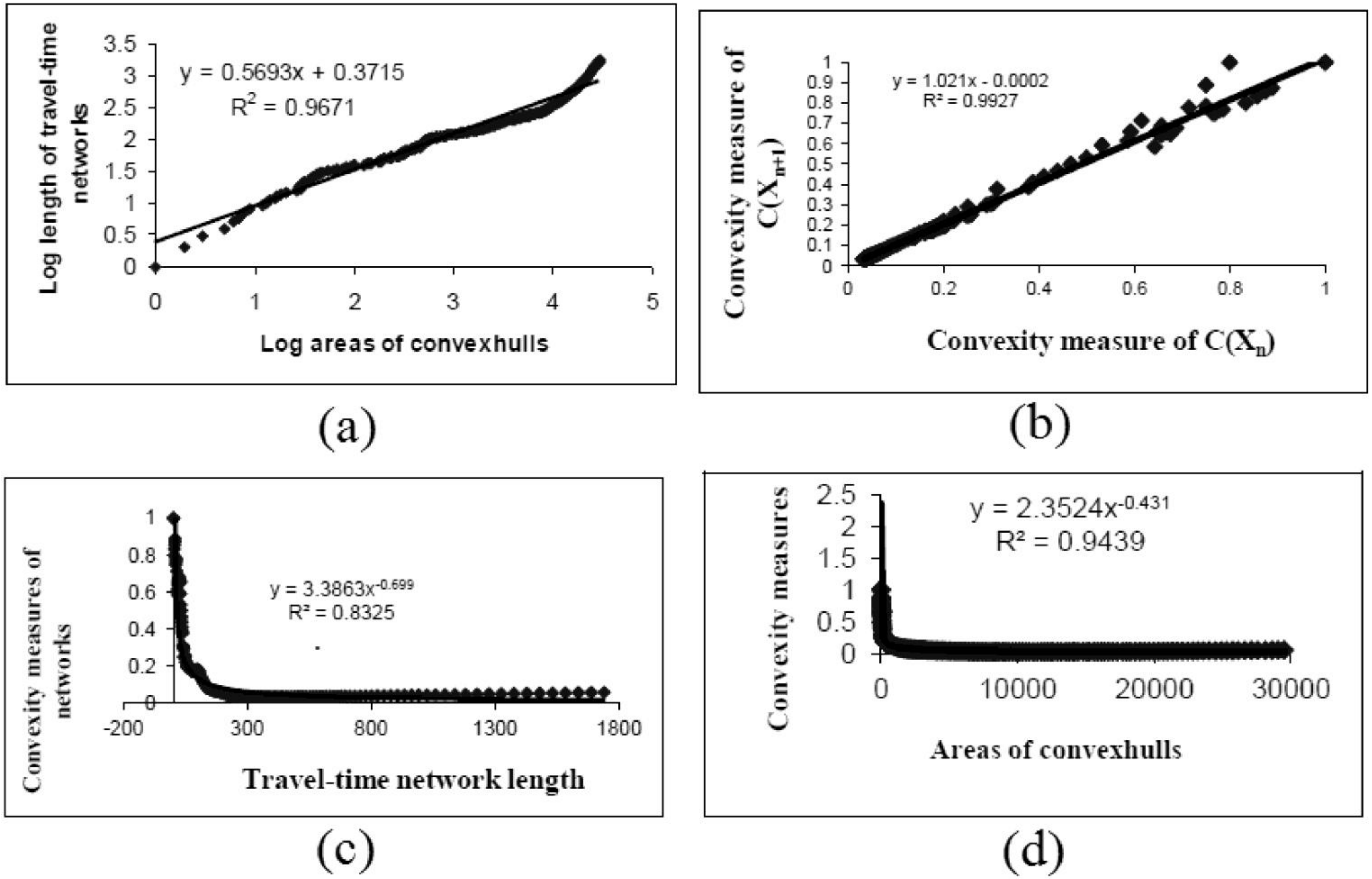
(a)

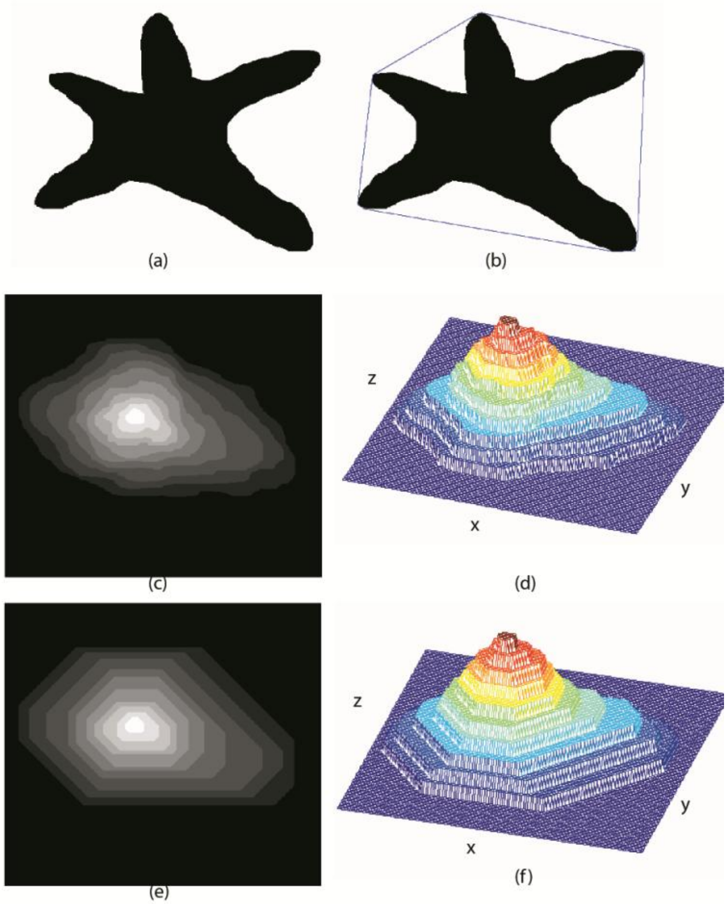


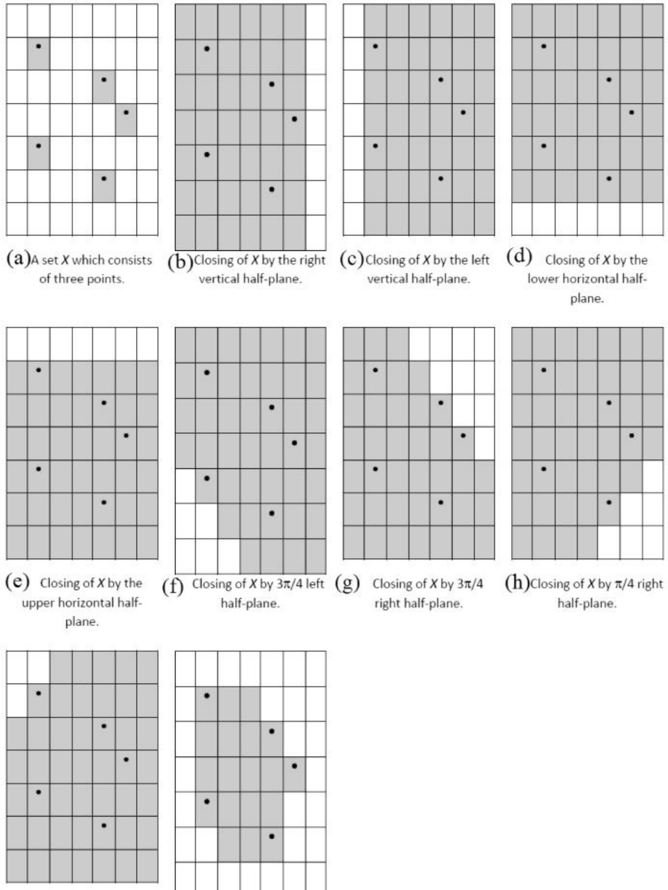
(b)



(a) (b) (c) (d) (e) (f)
(g) (h) (i) (j) (k) (l)


y = 0.5693x + 0.3715
R2 = 0.9671
Log length of travel-time networks
Log areas of convexhulls
(a)
y = 1.021x - 0.0002
R² = 0.9927
Convexity measure of C(Xn+1)
Convexity measure of C(Xn)
(b)
y = 3.3863x-0.699
R² = 0.8325
Convexity measures of networks
Travel-time network length
(c)
y = 2.3524x-0.431
R² = 0.9439
Convexity measures
Areas of convexhulls
(d)


(a)
(b)
z
y
x
(c)
(d)
z
y
x
(f)
(e)



(a) A set *X* which consists of three points.
(b) Closing of *X* by the right vertical half-plane.
(c) Closing of *X* by the left vertical half-plane.
(d) Closing of *X* by the lower horizontal half-plane.
(e) Closing of *X* by the upper horizontal half-plane.
(f) Closing of *X* by 3π/4 left half-plane.
(g) Closing of *X* by 3π/4 right half-plane.
(h) Closing of *X* by π/4 right half-plane.
(i) Closing of *X* by π/4 left half-plane.
(j) Intersection of closings (b) to (h)

(a) Half-plane closing of subset of *f*

19	25	21	30	25
14	17	16	222	20
8	12	240	254	208
9	209	250	255	254
15	208	240	253	252

→ Direction of translation

(b) Previous value = 0 (init)
Maximum along line = 19
Current value = max(0,19)

19	25	21	30	25
14	17	16	222	20
8	12	240	254	208
9	209	250	255	254
15	208	240	253	252

(c) Previous value = 19
Maximum along line = 209
Current value = max(19,209)

19	25	21	30	25
19	17	16	222	20
19	12	240	254	208
19	209	250	255	254
19	208	240	253	252

2^{nd} translation

(d) Previous value = 209
Maximum along line = 250
Current value = max(209,250)

19	209	21	30	25
19	209	16	222	20
19	209	240	254	208
19	209	250	255	254
19	209	240	253	252

3^{rd} translation

(e) Previous value = 250
Maximum along line = 255
Current value = max(250,255)

19	209	250	30	25
19	209	250	222	20
19	209	250	254	208
19	209	250	255	254
19	209	250	253	252

4^{th} translation

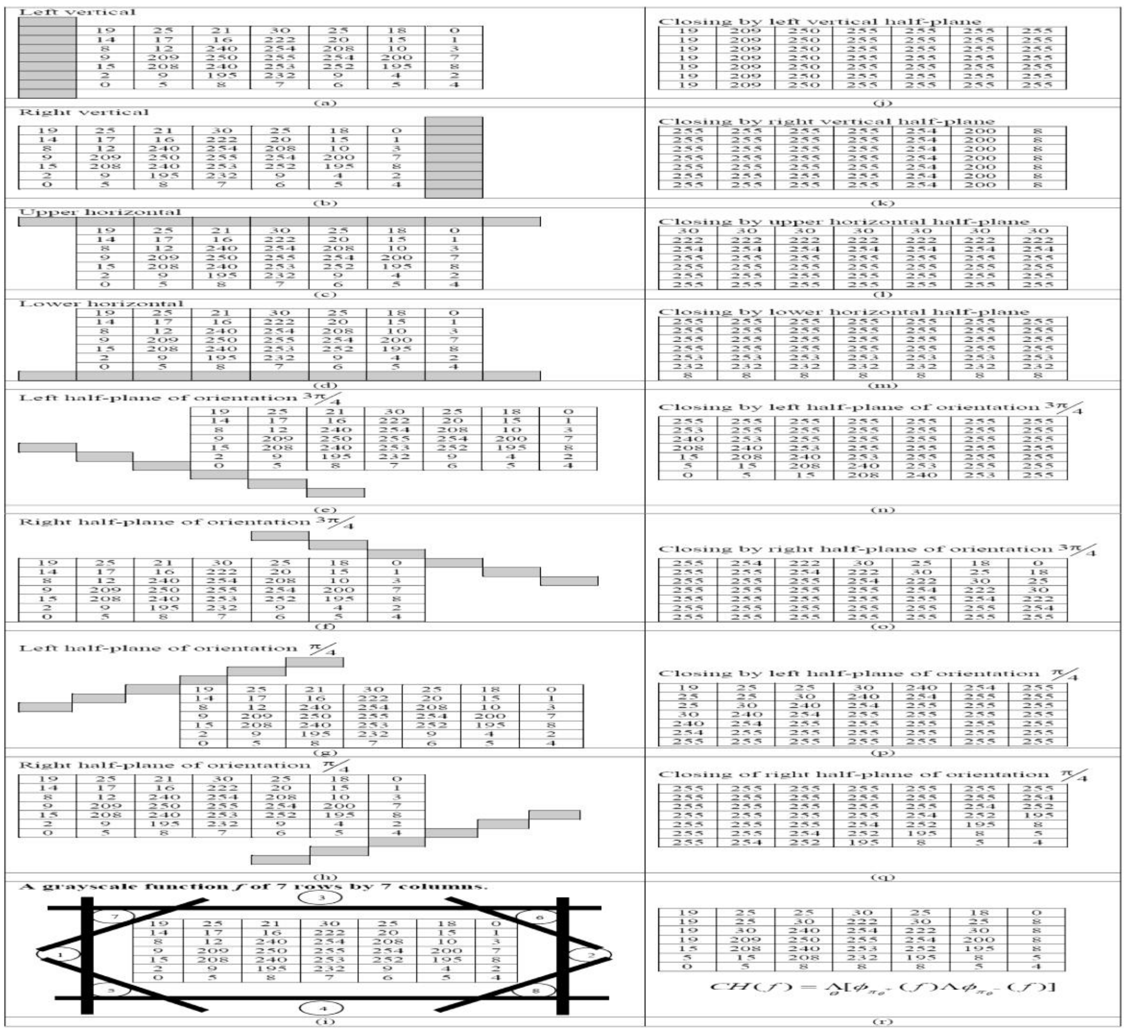
(f) Previous value = 255
Maximum along line = 254
Current value = max(255,254)

19	209	250	255	25
19	209	250	255	20
19	209	250	255	208
19	209	250	255	254
19	209	250	255	252

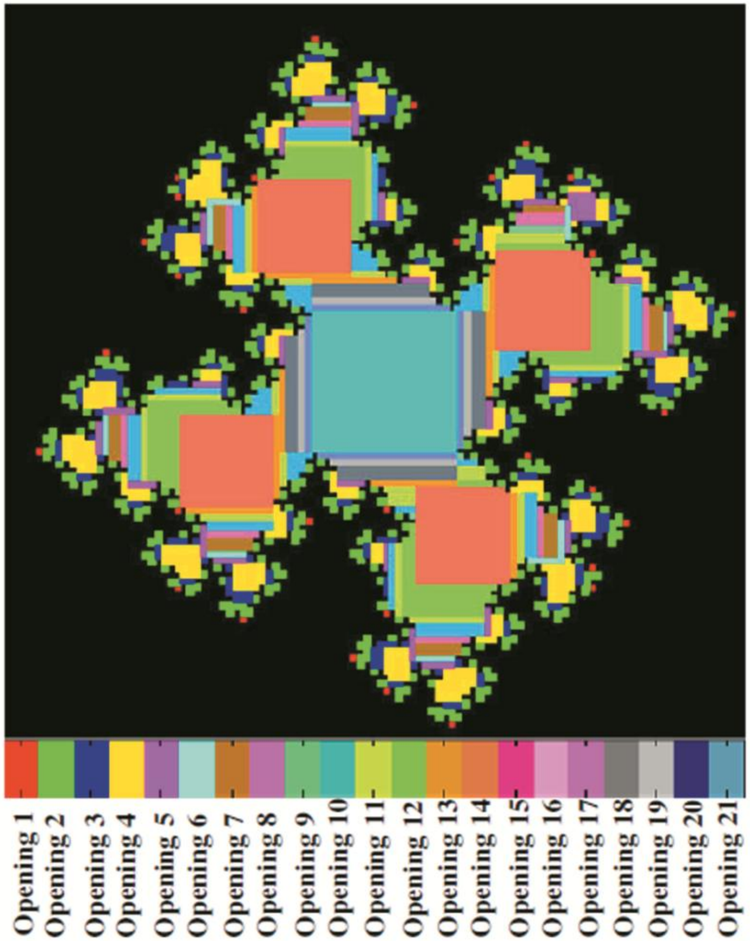
5^{th} translation

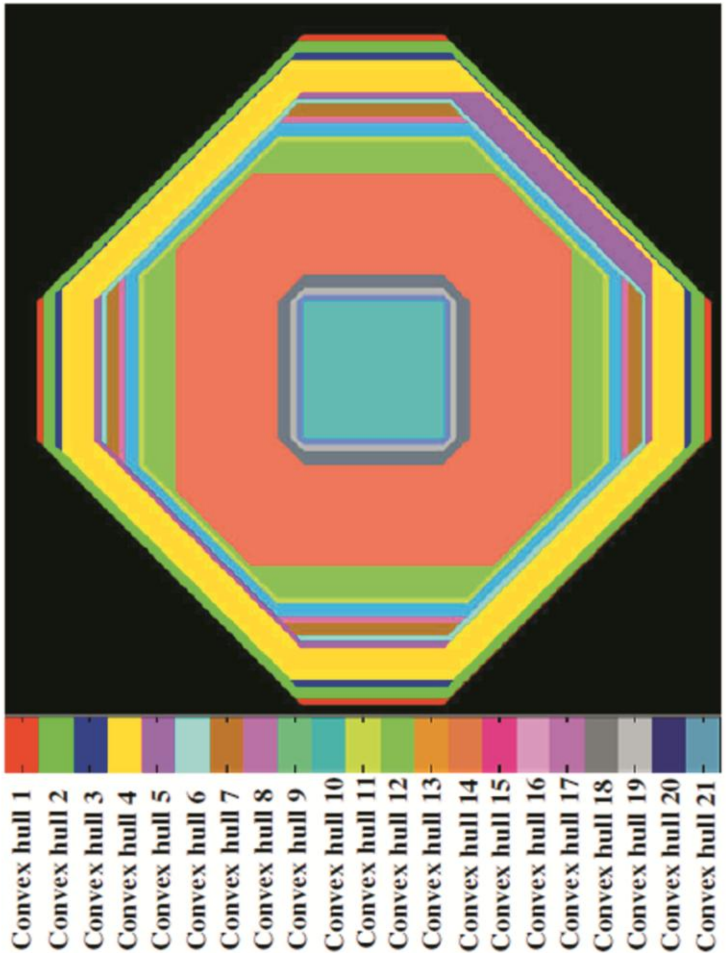
(g) Final result

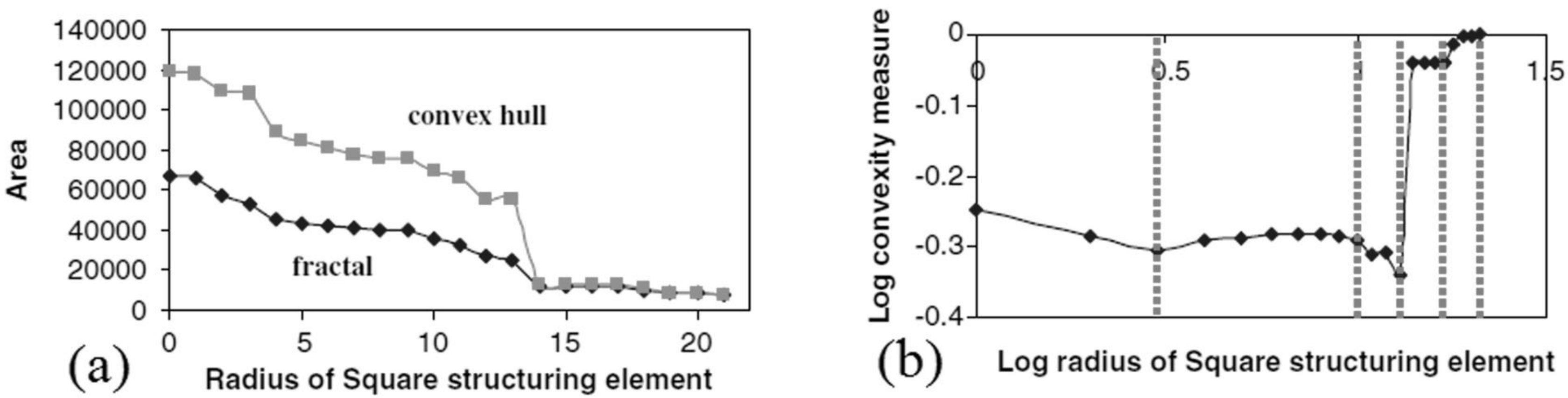
19	209	250	255	255
19	209	250	255	255
19	209	250	255	255
19	209	250	255	255
19	209	250	255	255



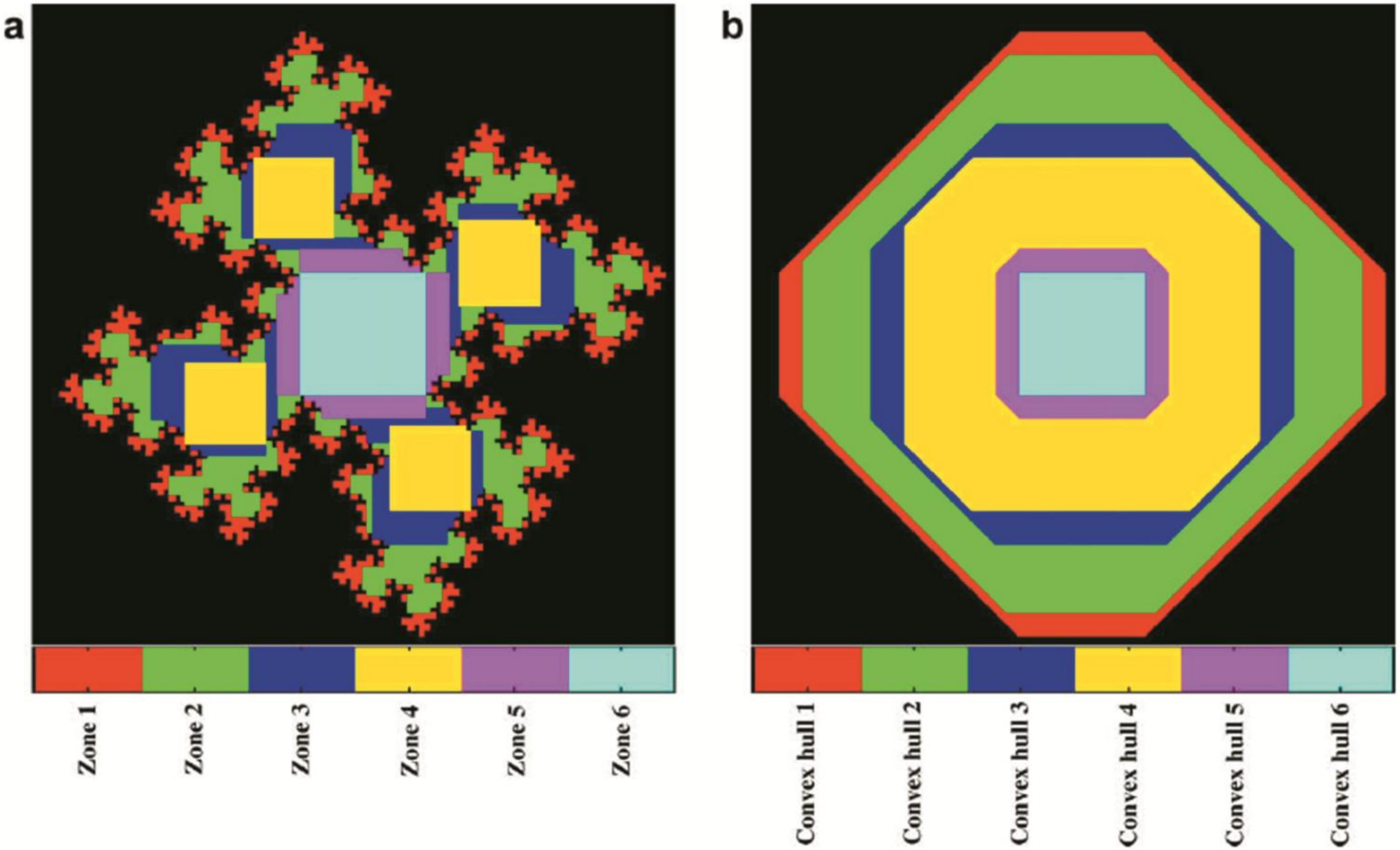
Left vertical
(a)
Right vertical
(b)
Upper horizontal
(c)
Lower horizontal
(d)
Left half-plane of orientation 3π/4
(e)
Right half-plane of orientation 3π/4
(f)
Left half-plane of orientation π/4
(g)
Right half-plane of orientation π/4
(h)
A grayscale function f of 7 rows by 7 columns.
(i)
Closing by left vertical half-plane
(j)
Closing by right vertical half-plane
(k)
Closing by upper horizontal half-plane
(l)
Closing by lower horizontal half-plane
(m)
Closing by left half-plane of orientation 3π/4
(n)
Closing by right half-plane of orientation 3π/4
(o)
Closing by left half-plane of orientation π/4
(p)
Closing of right half-plane of orientation π/4
(q)
CH(f) = ∧θ[φπθ+(f)∧φπθ−(f)]
(r)

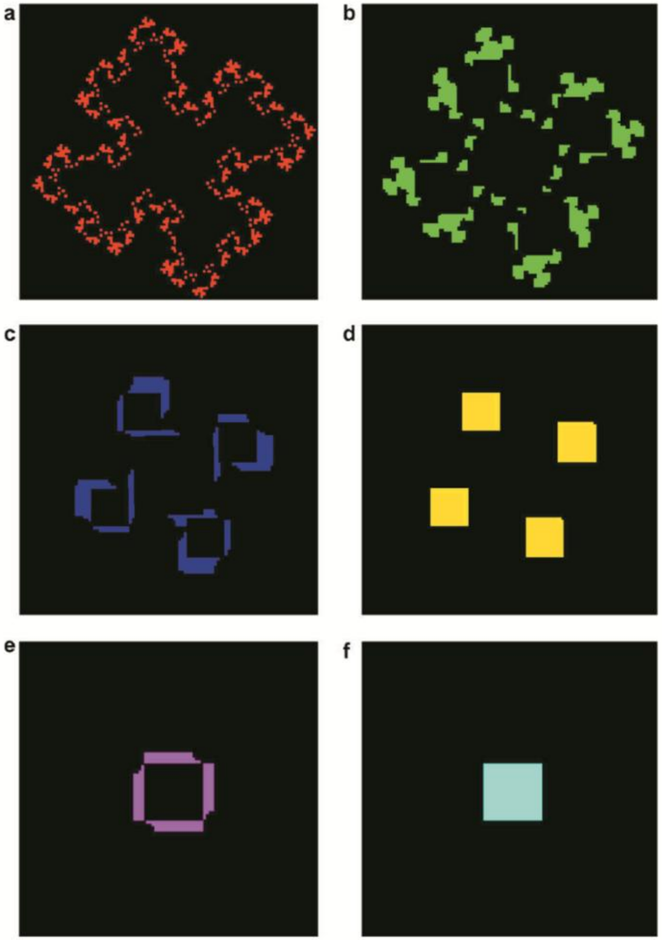

Opening 1
Opening 2
Opening 3
Opening 4
Opening 5
Opening 6
Opening 7
Opening 8
Opening 9
Opening 10
Opening 11
Opening 12
Opening 13
Opening 14
Opening 15
Opening 16
Opening 17
Opening 18
Opening 19
Opening 20
Opening 21

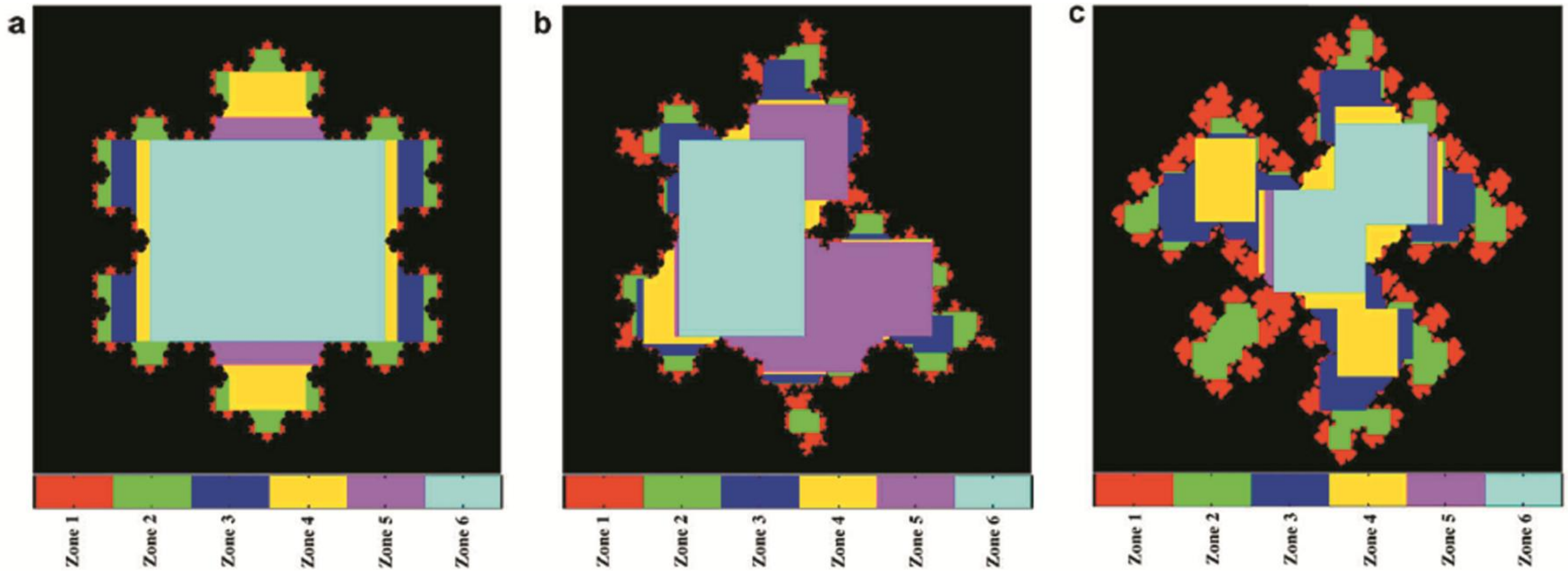

Convex hull 1
Convex hull 2
Convex hull 3
Convex hull 4
Convex hull 5
Convex hull 6
Convex hull 7
Convex hull 8
Convex hull 9
Convex hull 10
Convex hull 11
Convex hull 12
Convex hull 13
Convex hull 14
Convex hull 15
Convex hull 16
Convex hull 17
Convex hull 18
Convex hull 19
Convex hull 20
Convex hull 21



140000
120000
100000
80000
60000
40000
20000
0
Area
convex hull
fractal
0
5
10
15
20
(a)
Radius of Square structuring element
Log convexity measure
0
-0.1
-0.2
-0.3
-0.4
0
0.5
1
1.5
(b)
Log radius of Square structuring element


a
b
Zone 1
Zone 2
Zone 3
Zone 4
Zone 5
Zone 6
Convex hull 1
Convex hull 2
Convex hull 3
Convex hull 4
Convex hull 5
Convex hull 6


a
b
c
d
e
f


a
b
c
Zone 1
Zone 2
Zone 3
Zone 4
Zone 5
Zone 6

20	20	20	20	20	20	20	20	20	20	20
20	19	19	19	19	19	19	19	19	19	20
20	19	18	18	18	18	18	18	18	19	20
20	19	18	17	17	17	17	17	18	19	20
20	19	18	17	16	16	16	17	18	19	20
20	19	18	17	16	15	16	17	18	19	20
20	19	18	17	16	16	16	17	18	19	20
20	19	18	17	17	17	17	17	18	19	20
20	19	18	18	18	18	18	18	18	19	20
20	19	19	19	19	19	19	19	19	19	20
20	20	20	20	20	20	20	20	20	20	20

(a)

15	15	15	15	15	15	15	15	15	15	15
15	14	14	14	14	14	14	14	14	14	15
15	14	13	13	13	13	13	13	13	14	15
15	14	13	12	12	12	12	12	13	14	15
15	14	13	12	11	11	11	12	13	14	15
15	14	13	12	11	10	11	12	13	14	15
15	14	13	12	11	11	11	12	13	14	15
15	14	13	12	12	12	12	12	13	14	15
15	14	13	13	13	13	13	13	13	14	15
15	14	14	14	14	14	14	14	14	14	15
15	15	15	15	15	15	15	15	15	15	15

(b)

1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	1	0	0
0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0
0	0	0	1	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	1

(c)

1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1

(d)

20	0	0	0	0	0	0	0	0	0	20
0	19	0	0	0	0	0	0	0	19	0
0	0	18	0	0	0	0	0	18	0	0
0	0	0	17	0	0	0	17	0	0	0
0	0	0	0	16	0	16	0	0	0	0
0	0	0	0	0	15	0	0	0	0	0
0	0	0	0	16	0	16	0	0	0	0
0	0	0	17	0	0	0	17	0	0	0
0	0	18	0	0	0	0	0	18	0	0
0	19	0	0	0	0	0	0	0	19	0
20	0	0	0	0	0	0	0	0	0	20

(e)

15	0	0	0	0	0	0	0	0	0	15
0	14	0	0	0	0	0	0	0	14	0
0	0	13	0	0	0	0	0	13	0	0
0	0	0	12	0	0	0	12	0	0	0
0	0	0	0	11	0	11	0	0	0	0
0	0	0	0	0	10	0	0	0	0	0
0	0	0	0	11	0	11	0	0	0	0
0	0	0	12	0	0	0	11	0	0	0
0	0	13	0	0	0	0	0	13	0	0
0	14	0	0	0	0	0	0	0	14	0
15	0	0	0	0	0	0	0	0	0	15

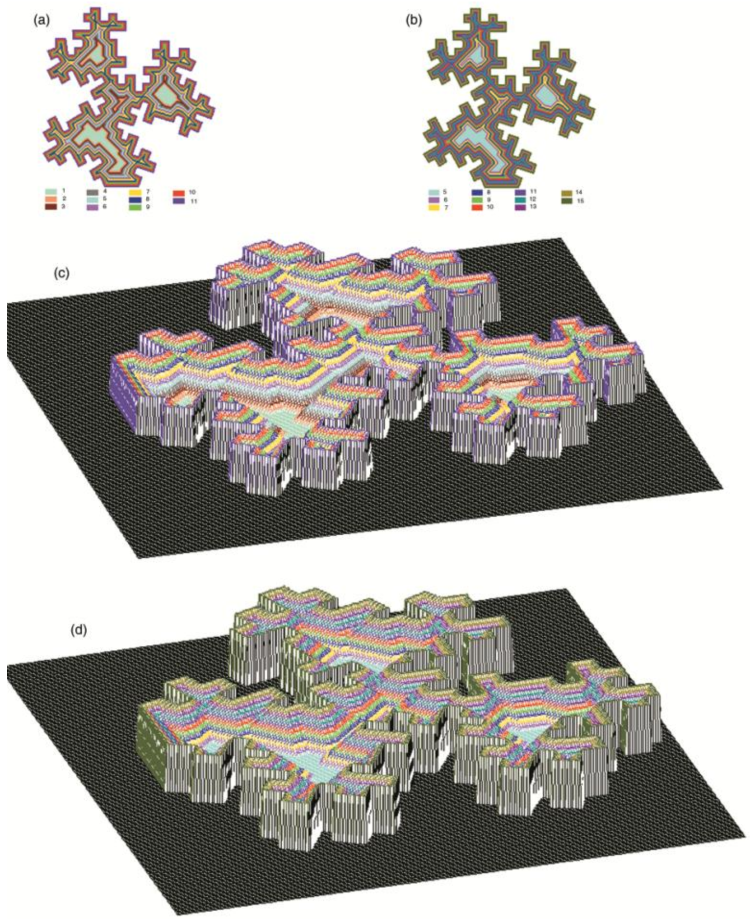
(f)

20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20
20	20	20	20	20	20	20	20	20	20	20

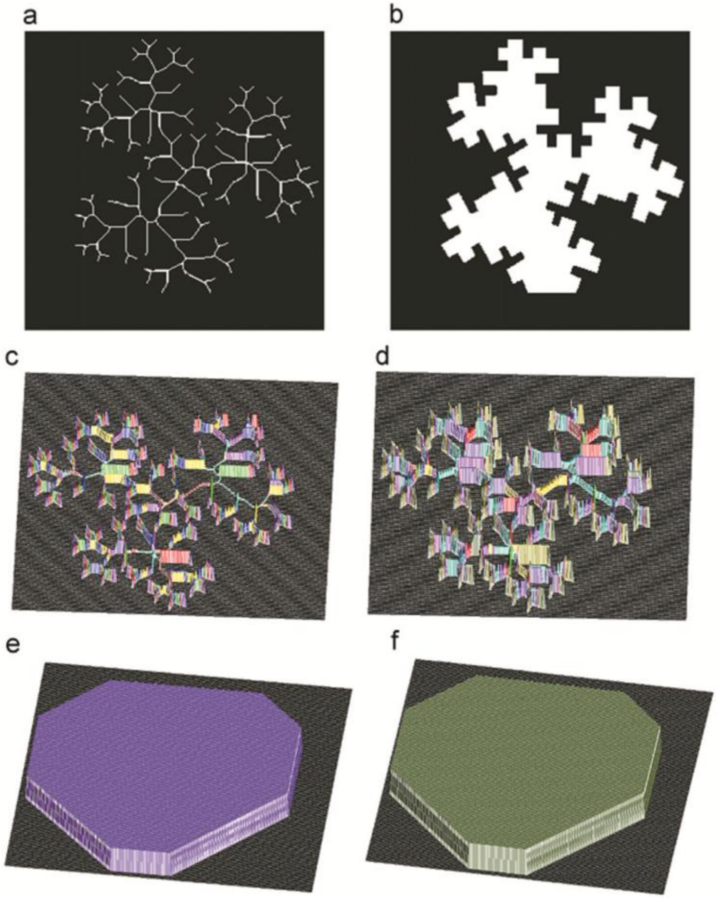
(g)

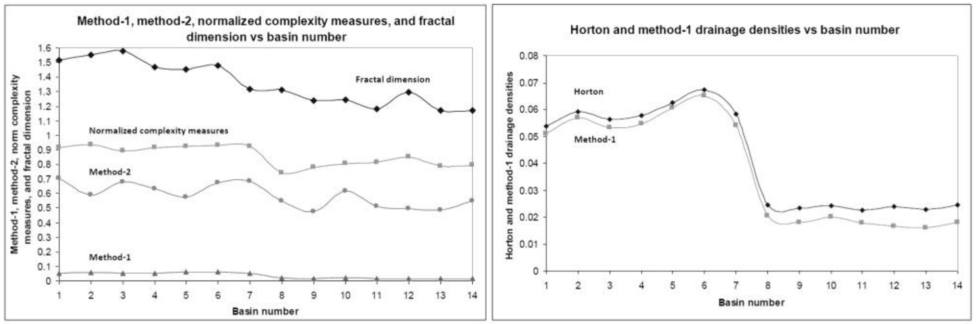
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15

(h)



(a)
(b)
(c)
(d)


a
b
c
d
e
f


Method-1, method-2, normalized complexity measures, and fractal dimension vs basin number
Method-1, method-2, norm complexity measures, and fractal dimension
Fractal dimension
Normalized complexity measures
Method-2
Method-1
Basin number
Horton and method-1 drainage densities vs basin number
Horton and method-1 drainage densities
Horton
Method-1
Basin number
(a) (b)