

LOGISTIC MAPS AND BIFURCATION THEORY

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Attractors in Predictions

- Data corresponding to systems with simple behaviors can be represented in the form of *simple attractors*
- Data corresponding to systems with strange behaviors can be represented in the form of *strange attractors*
- *Simple Attractors* possess low dimensionality, whereas *Strange Attractors* possess high dimensionality
- The predicting predictability of the systems with low dimensional attractors is rather straightforward.
- The predicting predictability of the systems with high dimensional attractors is complex.

BIG Question(s)

- ∞ Do we have information thus retrieved precisely from DATA (whether Small or Big) that leads to construction of system-specific *attractor*?
- ∞ How big is the data that we require to construct such an attractor? An ad hoc answer is another question: What is the system that we are targeting?
- ∞ Can we categorize the systems as ‘soft’ and ‘hard’.
 - Soft – *simple attractor* – prediction possible
 - Hard – *strange attractor* – prediction (locally) is possible

Some Roots, of 'Predictive Analytics', that I know

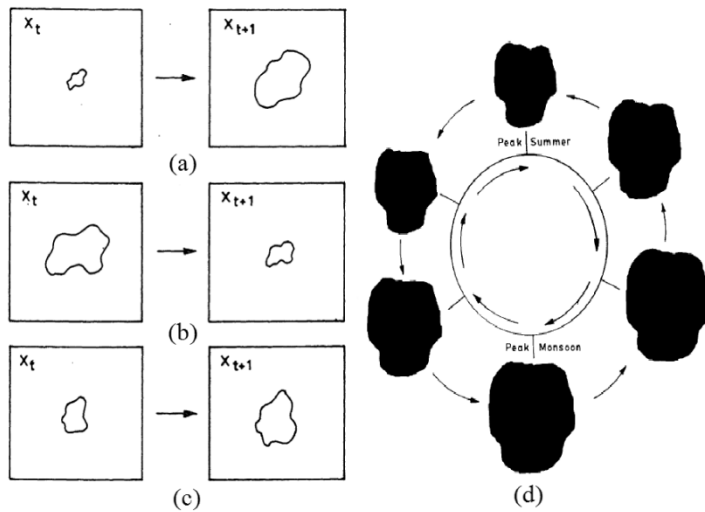
☞ Some philosophical speculations!

Wonderful Recipe!

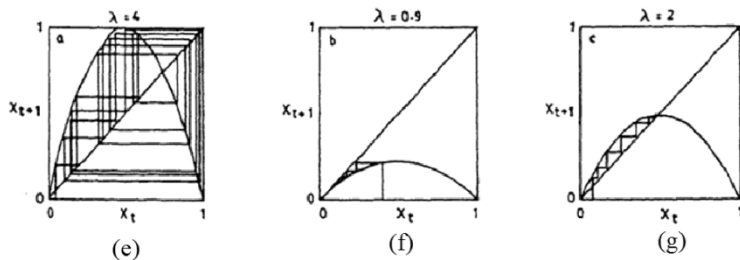
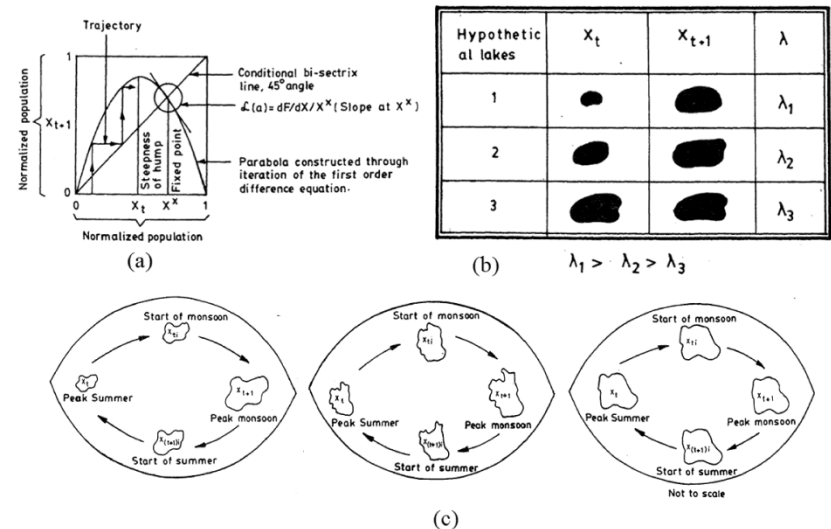
- ☞ Rise is to fall.
- ☞ Make it larger when it is small
- ☞ Make it small when it is large
- ☞ Follow above in a nonlinear fashion to simulate several dynamical processes mimicking realistic dynamical processes.
- ☞ Numerous controls control a system (be it a business-systems or a natural system).
- ☞ What is(are) the strength(s) of control(s) that control(s) the control of another system: Systems behaviors are highly coupled.

Examples of Attractors: Toy Models

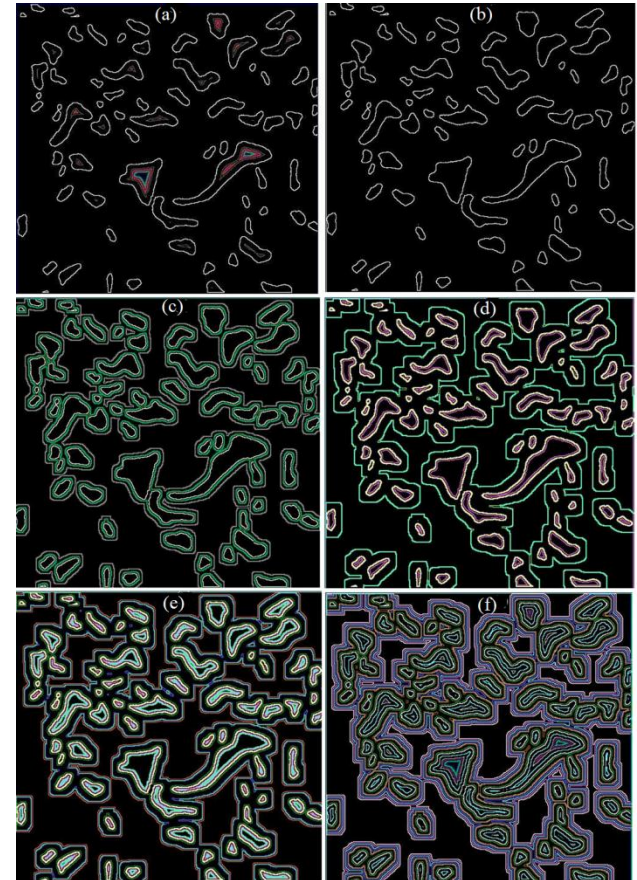
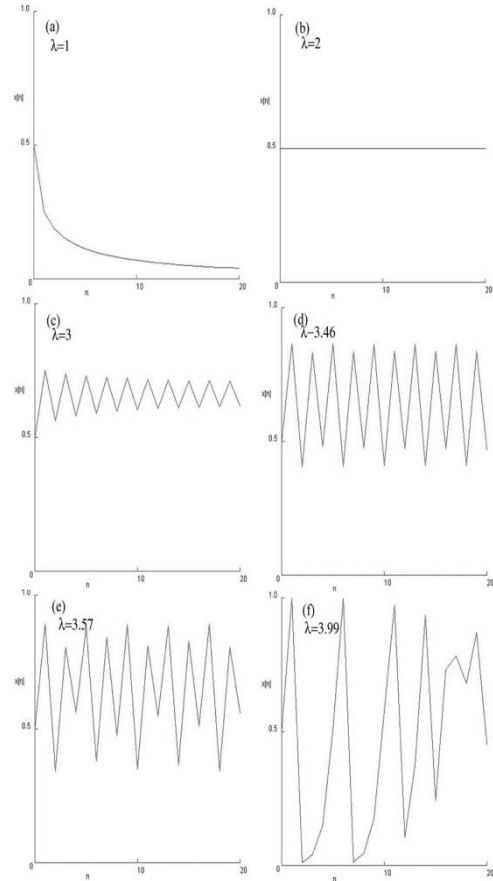
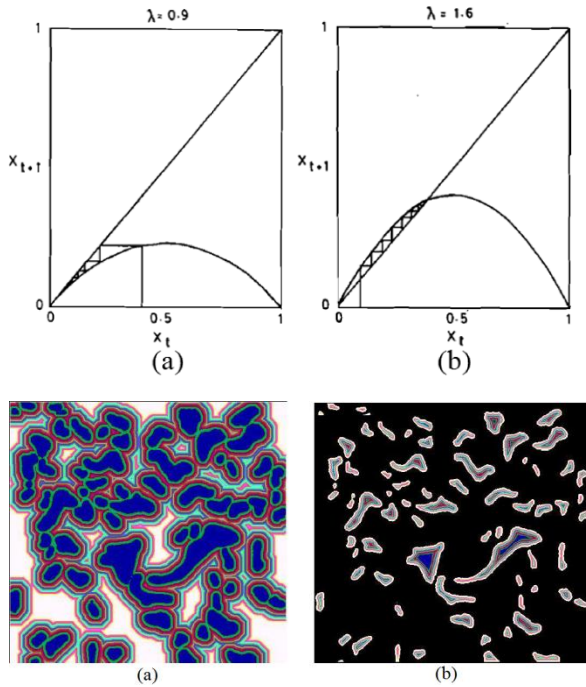
Numerical Data



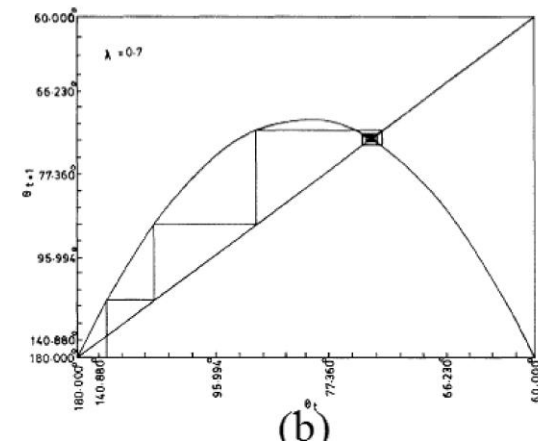
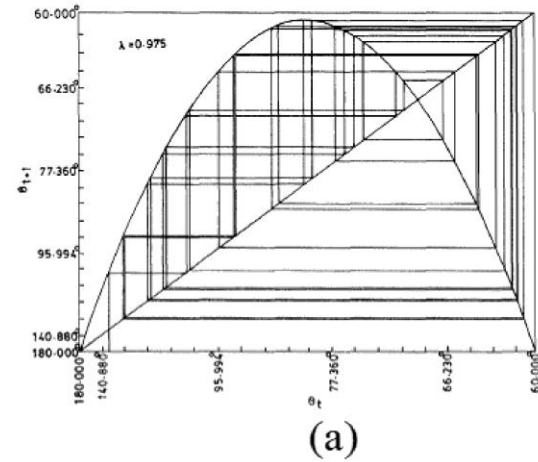
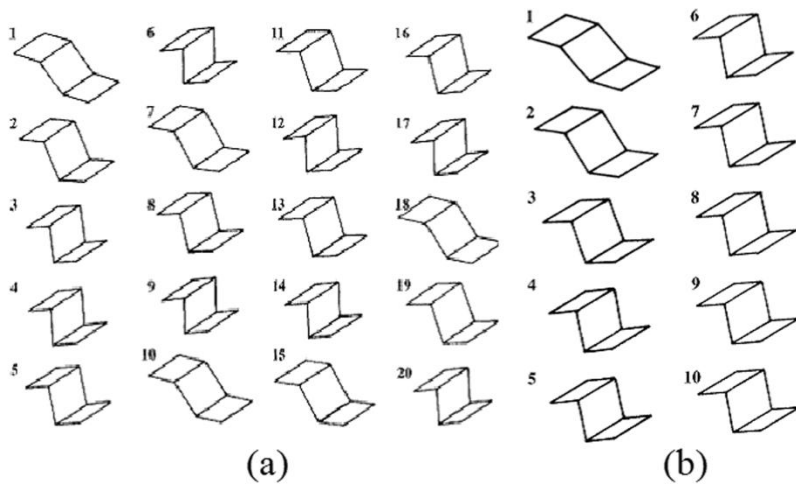
Attractors



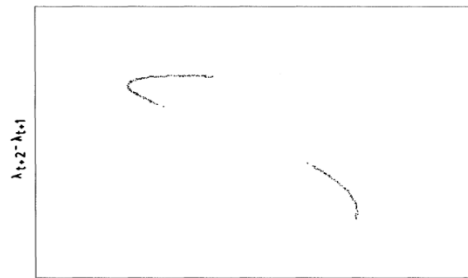
Behavior of Lakes



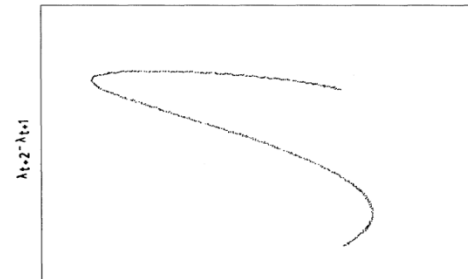
Behavior of Folds



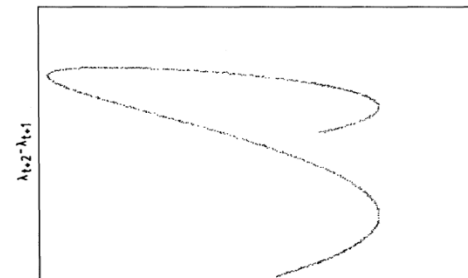
Attractors as Phase Space Maps



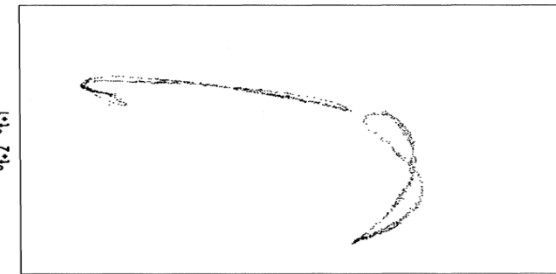
$\lambda_{t+1} - \lambda_t$
(a)



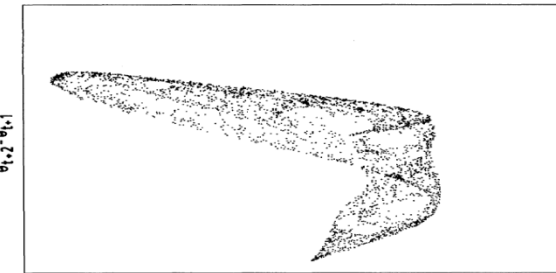
$\lambda_{t+1} - \lambda_t$
(b)



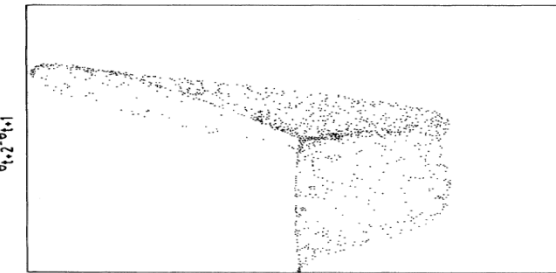
$\lambda_{t+1} - \lambda_t$
(c)



$\theta_{t+1} - \theta_t$
(a)



$\theta_{t+1} - \theta_t$
(b)



$\theta_{t+1} - \theta_t$
(c)

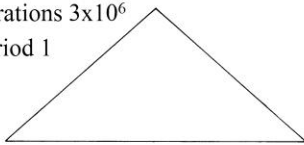
Behavior of Sand Dunes

(a) Initial sand dune profile



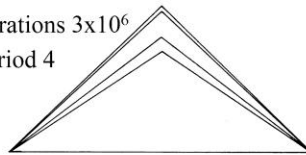
(b) $\lambda = 3.0$

Iterations 3×10^6
Period 1



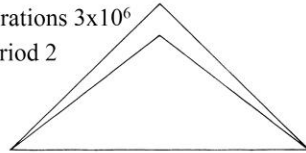
(d) $\lambda = 3.569$

Iterations 3×10^6
Period 4



(c) $\lambda = 3.46$

Iterations 3×10^6
Period 2



(e) $\lambda = 3.57$

Iterations 3×10^6
Period 8

