MATHEMATICAL MORPHOLOGY IN GEOSCIENCES AND GISci

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III. Mathematical Morphology in GISci

Spatial Interpolations

Spatial Reasoning

- Strategic set identification
- Directional Spatial Relationship
- Spatial-Interactions
- Point-to-Polygon Conversion

III.I. Spatial Interpolations

VISUALIZATION OF SPATIO-TEMPORAL BEHAVIOUR OF DISCRETE MAPS VIA GENERATION OF RECURSIVE MEDIAN ELEMENTS

Outline

Mathematical Morphological Transformations employed include:

Hausdorff Dilation, Hausdorff Erosion, Morphological Median Element Computation, and Morphological Interpolation.

Objectives

To show relationships between the layers depicting noise-free phenomenon at two time periods.

To relate connected components of layers of two time periods via FOUR possible categories of spatial relationships of THREE groups.

To propose a framework to generate recursive interpolations via median set computations.

To demonstrate the validity of the framework on epidemic spread.

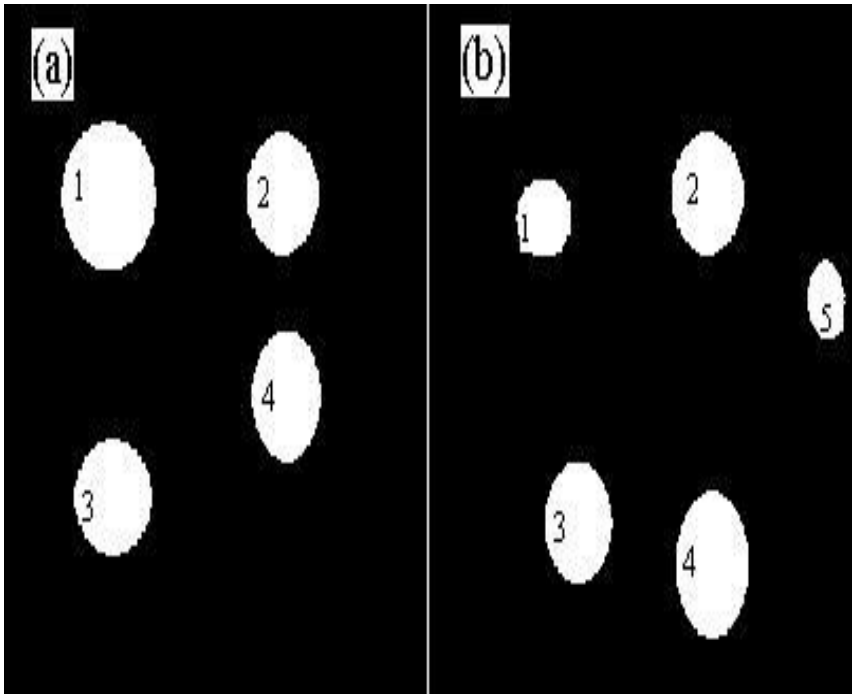
THREE Groups and FOUR Categories??

Three groups are conceived by checking the intersection properties between the corresponding connected components.

Four categories under the above three groups are visualized via logical relationships and Hausdorff erosion and Hausdorff dilation distances.

What are these Hausdorff distances?

What basics do we require to know to compute these distances?



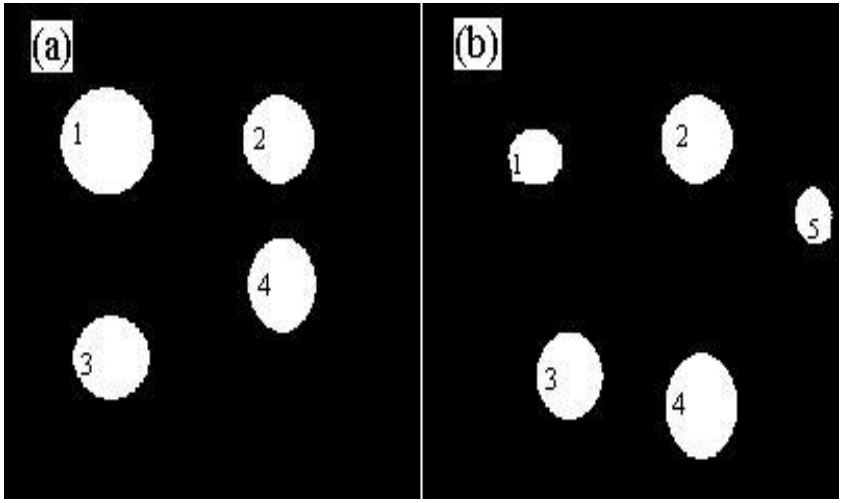
Spatial Relationships Between Sets and Their Categorization

- *Ordered sets*.

semi-ordered sets, if subsets of X^t (resp. X^{t+1}) are only partially contained in the other set X^{t+1} (resp. X^t).

Whereas, (X^t) and (X^{t+1}) are considered as *disordered sets* if there exists an empty set while taking the intersection of (X^t) and (X^{t+1}) (or) of their corresponding subsets.

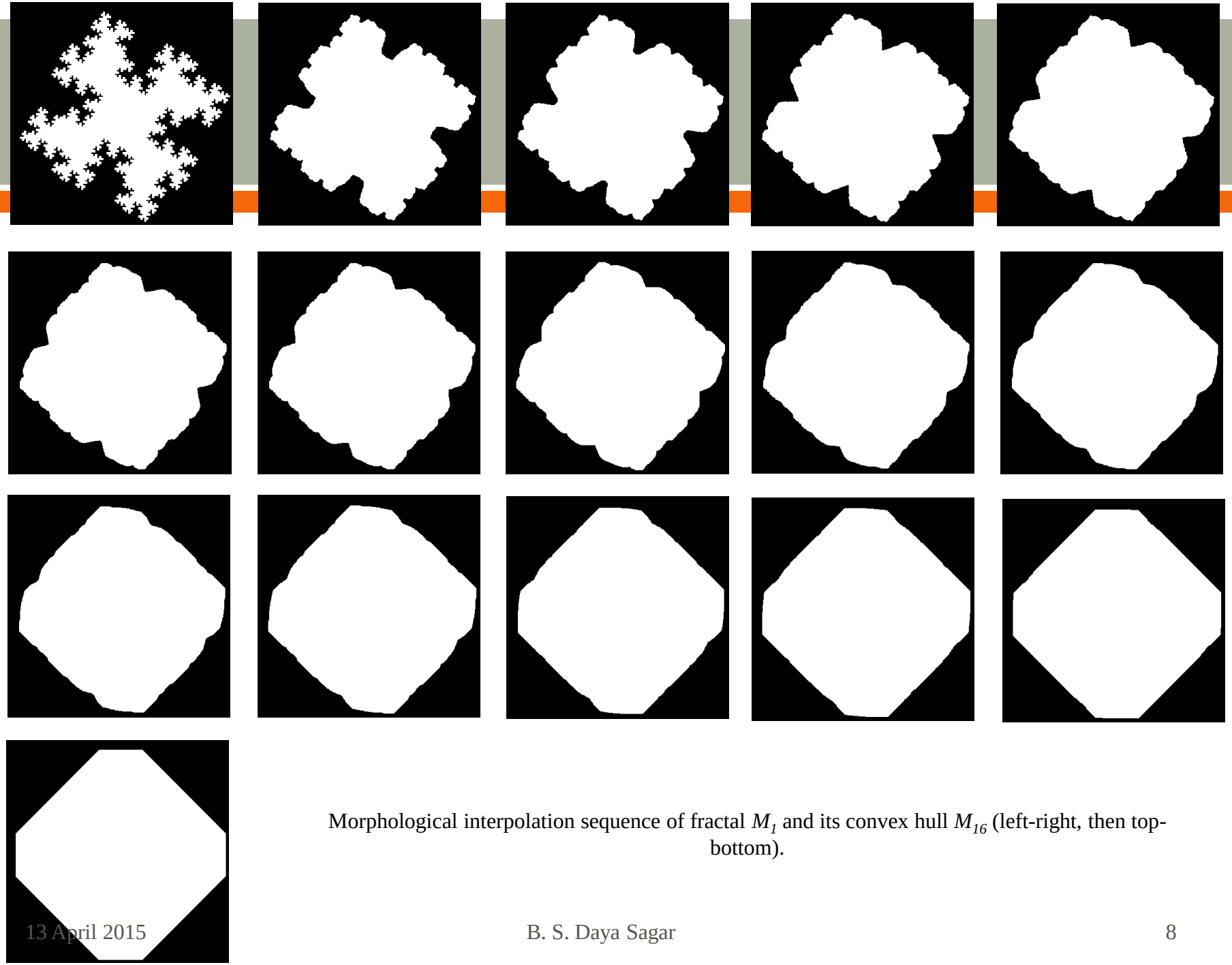
Description of categories via logical relations



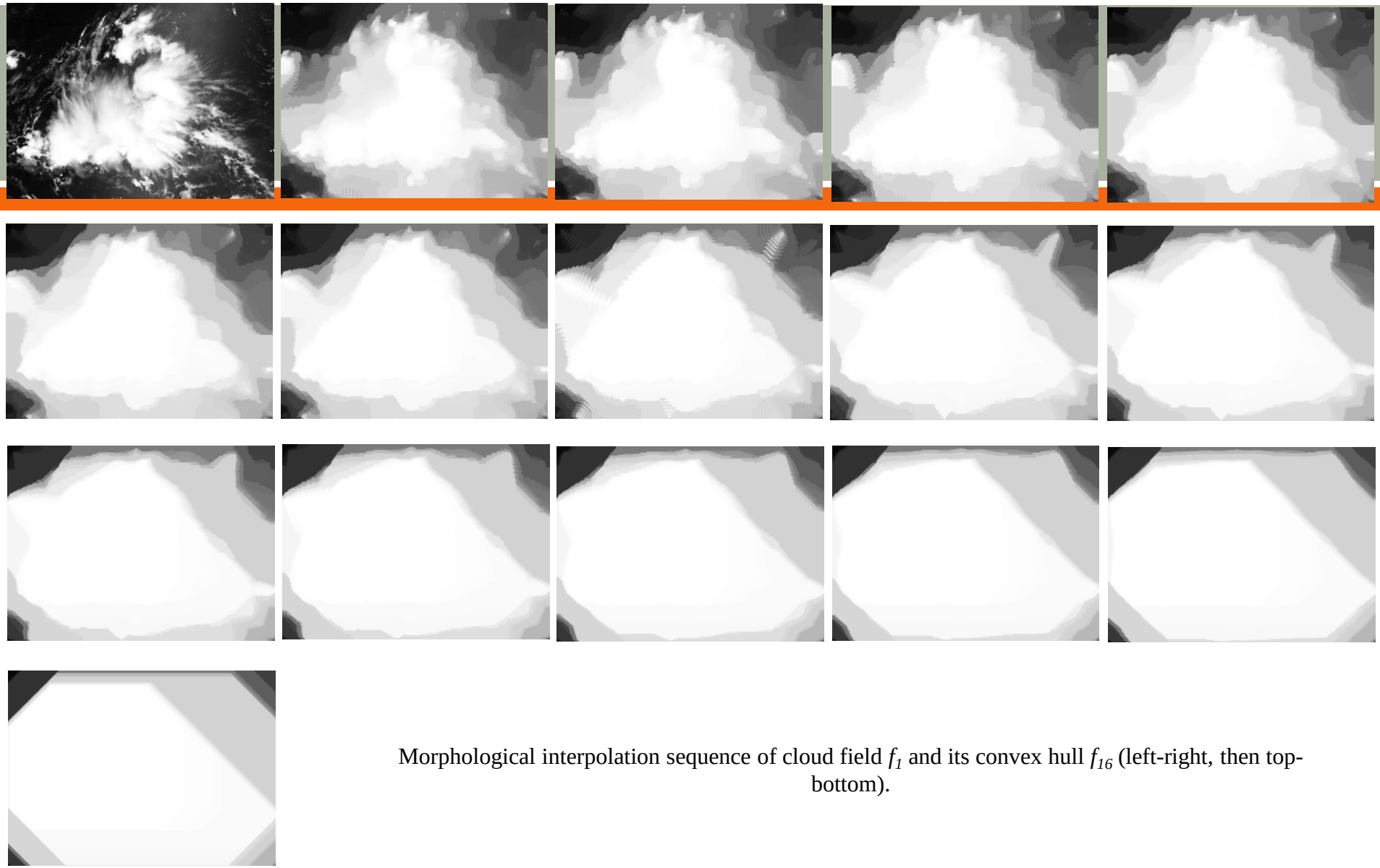
Categories via Hausdorff Erosion and Dilation Distances

TABLE 1. CATEGORY-WISE HAUSDORFF DISTANCES

Group	Category	$\sigma\left(X_i^t, X_i^{t+1}\right)$	$\rho\left(X_i^t, X_i^{t+1}\right)$
I	1	0	0
I	2	≥ 1	≥ 1
II	3	Does not exist	≥ 1
III	4	Does not exist	Does not exist



Morphological interpolation sequence of fractal M_1 and its convex hull M_{16} (left-right, then top-bottom).



Morphological interpolation sequence of cloud field f_1 and its convex hull f_{16} (left-right, then top-bottom).

Interpolated Sequence of Lakes' Data of Two Seasons

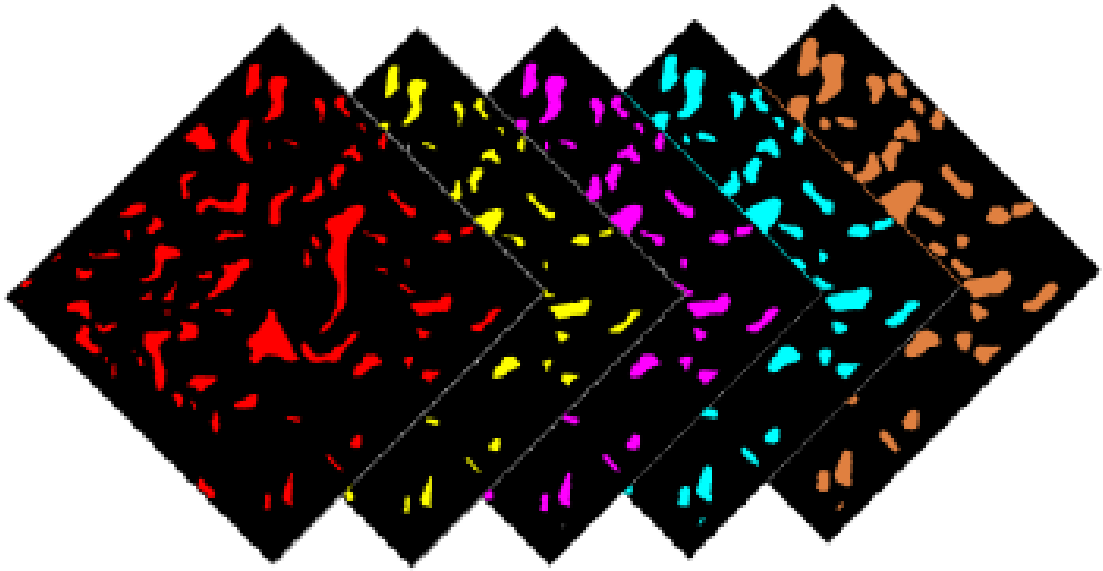
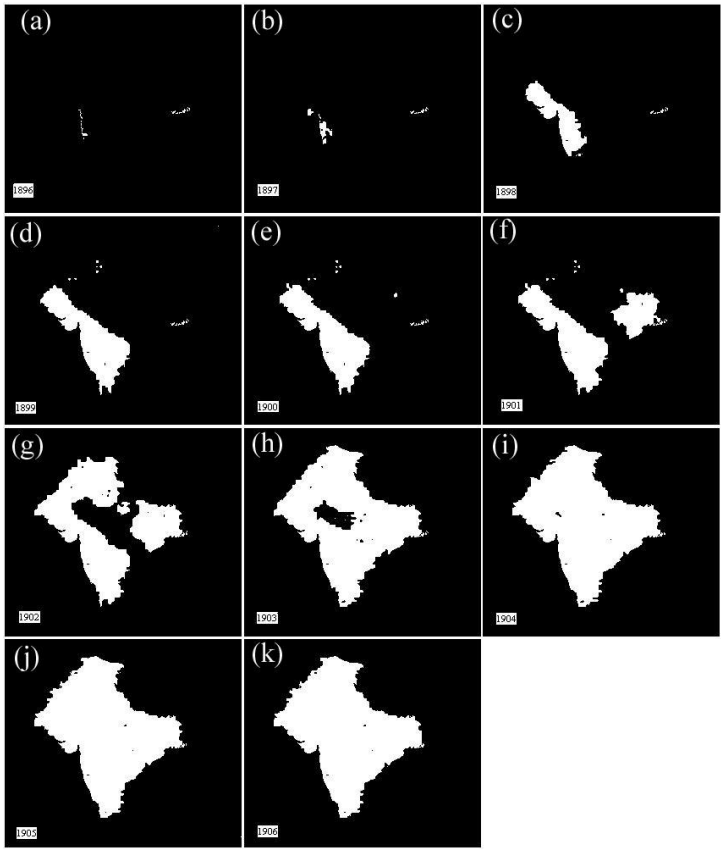
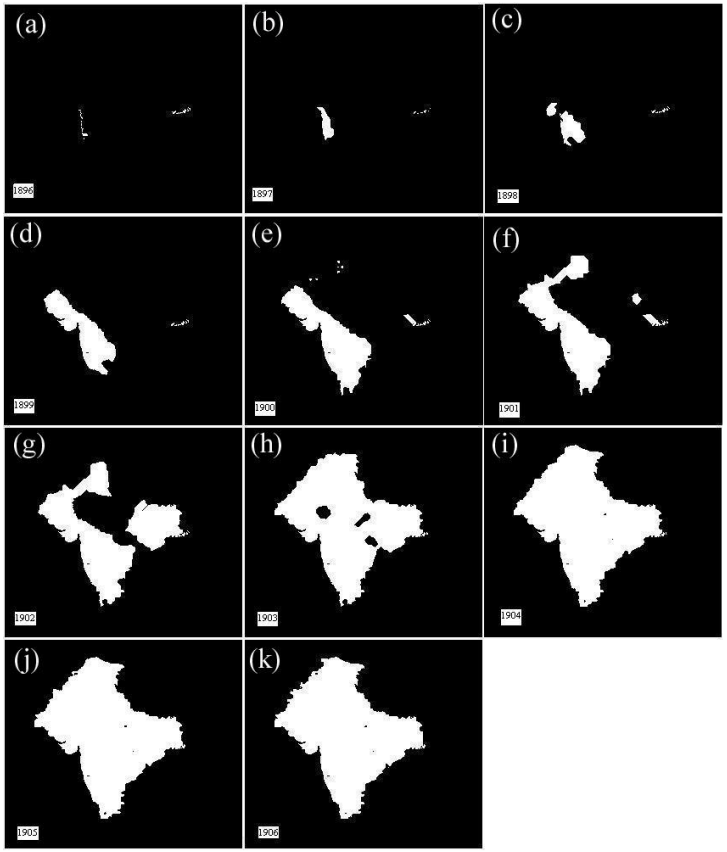


Fig. 4. A sequence of interpolated sets (slices) in between the two input slices shown in Figs. 3a, b. Equations 8(a) and 14 are used to recursively generate the interpolated slices. The layer depicting water bodies with magenta color is the median set shown in Fig. 3c.

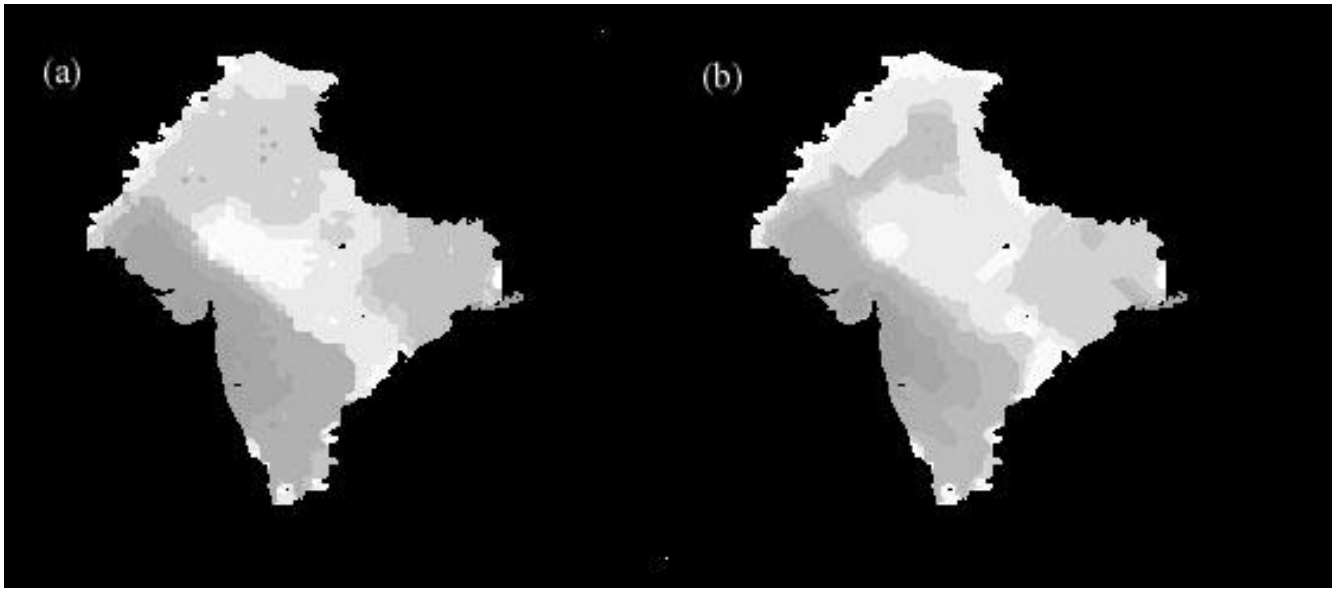
Observed and Interpolated Epidemic Spread Maps

http://www.isibang.ac.in/~bsdsagar/AnimationOfEpidemicSpread.avi





Observed and Interpolated Sequences



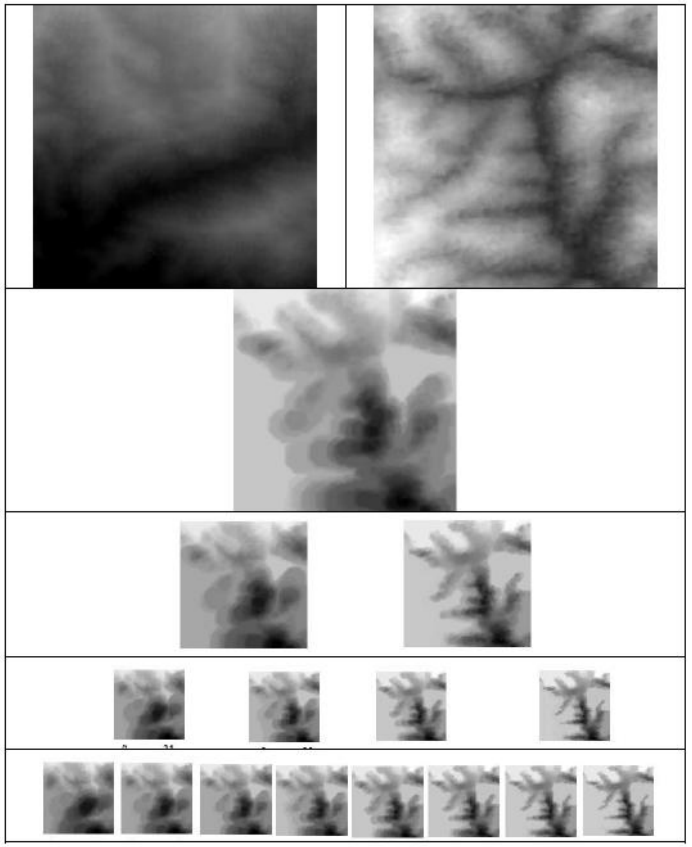
Earth Surface Transformation

Hierarchical Morphological Interpolation between landscape functions, say, f_1 and f_{256}.

f_1 f_{256}

f_{128}

f_{64} f_{192}

f_{32} f_{96} f_{160} f_{224}

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