Watershed Transformation:
Morphological Segmentation Algorithm

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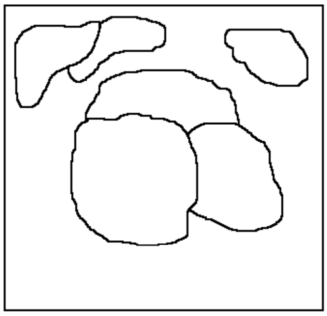
Contents

- Introduction
- Preliminaries
  - Binary Morphological operators
  - Grayscale Morphological operators
  - Distance transforms
- Watershed transformation
  - Algorithm and results
- Conclusion

An example of segmentation

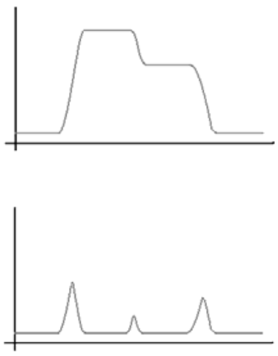
Original image



Desired output

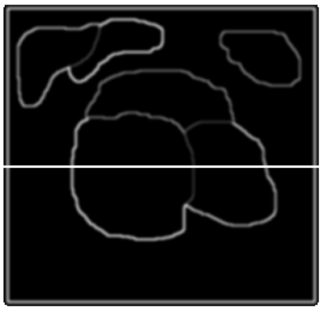
Finding gradient

Consider the row as shown

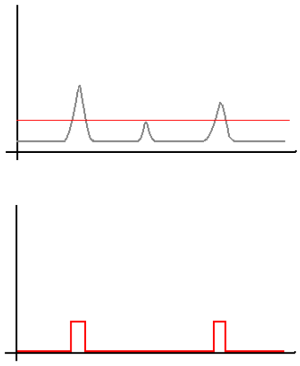


Intensity profile and gradient

Detecting boundary



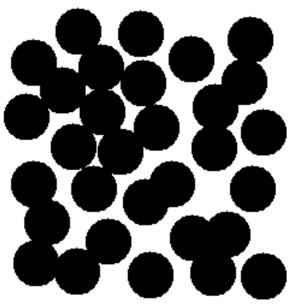
Consider the row as shown



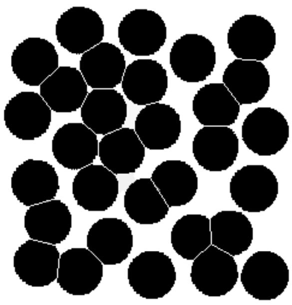
Applying threshold and output

Note the problem of such approach.

Segmenting binary objects



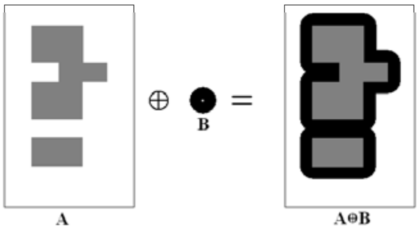
Original image



Desired output

Binary dilation

Expands the objects.

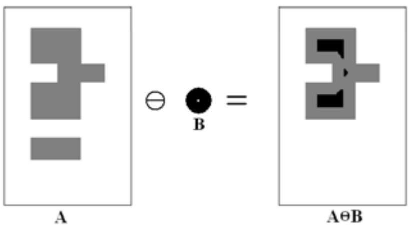


$$A \oplus B = \{a + b \mid a \in A, b \in B\} = \bigcup_{a \in A} B_a = \bigcup_{b \in B} A_b$$

Properties: Commutative, associative,
distributive (over union), increasing

Binary erosion

Shrinks the objects.



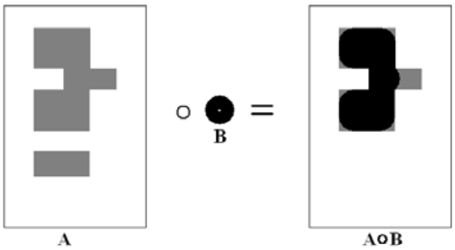
$$A \Theta B = \{p \mid B_p \subseteq A\} = \bigcap_{p \in B} A_{-p}$$

Properties: Increasing, distributive (over intersection)

Dilation and erosion are dual.

Binary opening

Removes objects or parts of it that cannot fit in SE.

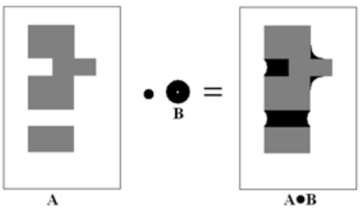


$$A \circ B = \left(A \Theta B\right) \oplus B$$

Properties: Increasing, idempotent, anti-extensive. It is a filter.

Binary closing

Appends to objects those parts of background that cannot fit in SE.



$$A \bullet B = \left(A \oplus B\right) \Theta B$$

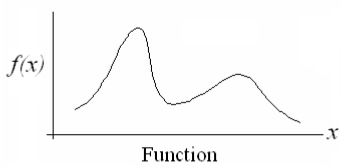
Properties: Increasing, idempotent, extensive. It is a filter.

Opening and closing are dual.

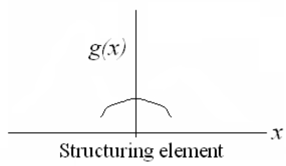
Grayscale morphology

- Useful for handling image information
- May be defined by extending binary morpholo-gical operators to higher domain
- For example, a one-dimensional function f(r) of r may be viewed as a set of points or shape in two-dimensional coordinate system.
- Similarly, an image function f(r,c) may be extended to 3D as (r, c, f(r,c)).

Grayscale morphology: example

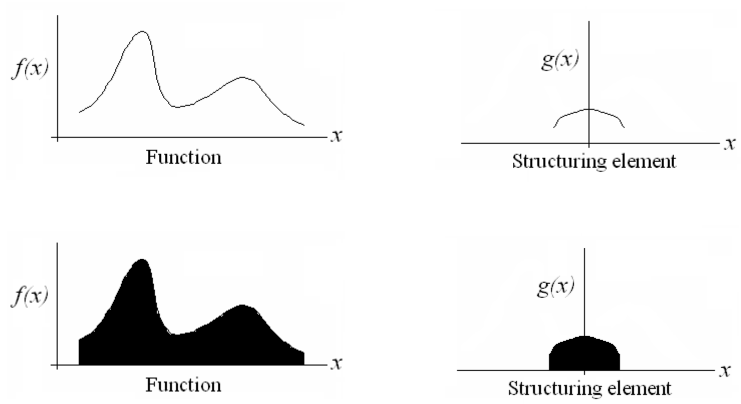


Function

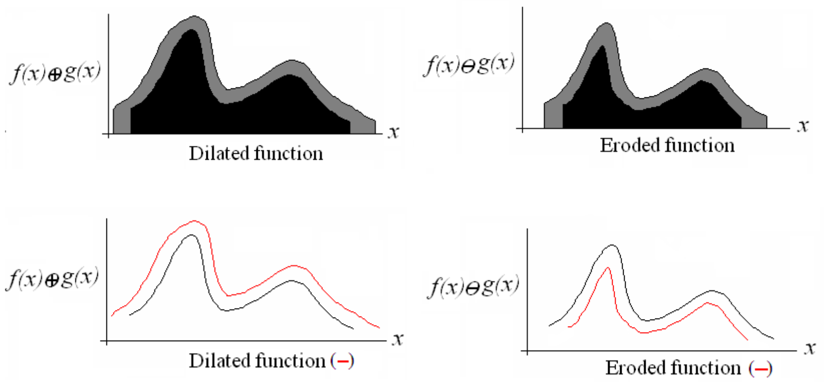


Structuring element

Grayscale morphology: example



Grayscale morphology: example



Grayscale morphology

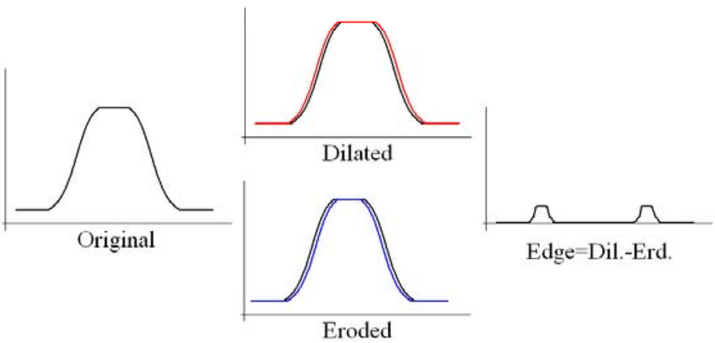
1. Dilation: $(g \oplus h)(r,c) = \max\limits_{(m,n)} \{ g(r-m, c-n) + h(m,n) \}$

2. Erosion: $(g \Theta h)(r,c) = \min\limits_{(m,n)} \{ g(r+m, c+n) - h(m,n) \}$

3.Opening: $(g \circ h)(r,c) = ((g \Theta h) \oplus h)(r,c)$

4. Closing: $(g \bullet h)(r,c) = ((g \oplus h) \Theta h)(r,c)$

Edge detection: example



$$g(r,c) = (f \oplus h)(r,c) - (f \Theta h)(r,c)$$

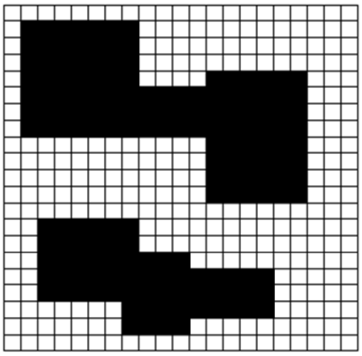
Edge detection: example

Original

Diff = Dilated - Eroded

Distance transform

- For a point $x \in A$, distance *dist(x)* is the distance of *x* from nearest point of A^c .
- *dist(.)* may be Euclidean, city-block or chessboard.
- Distance transform is a mapping of each pixel of a set to its distance value.



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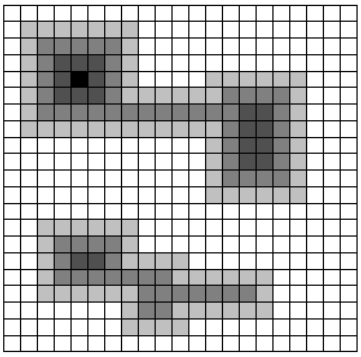
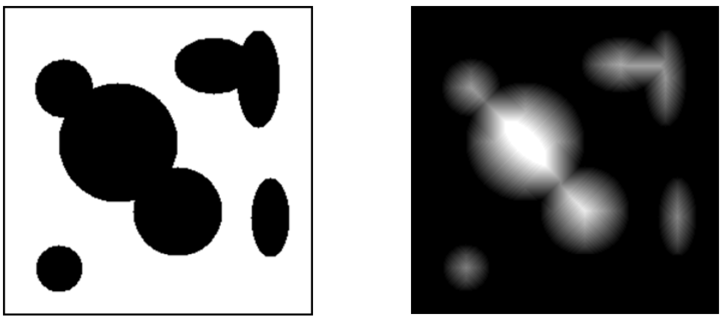
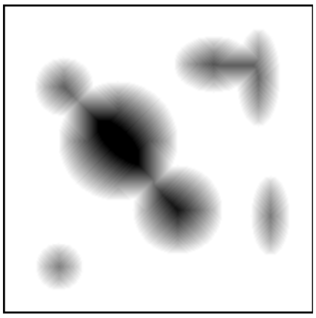
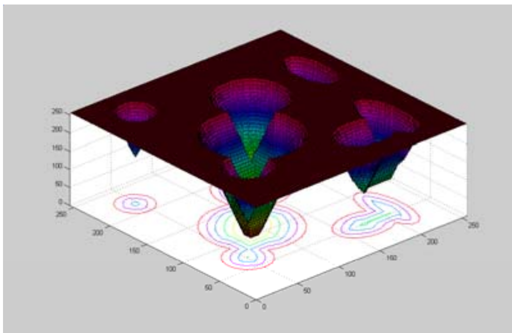


A simple example



DT and 3D surface





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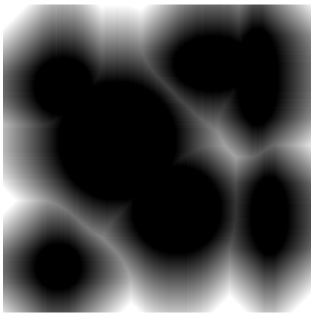

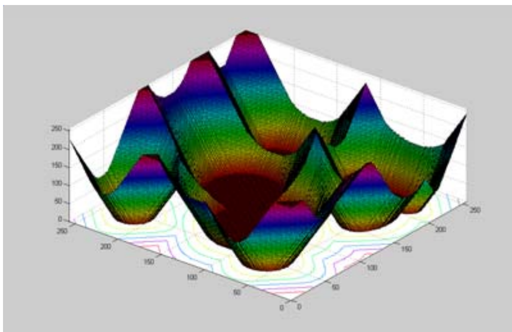

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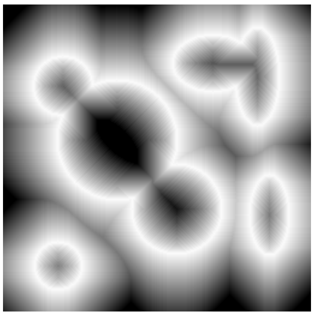
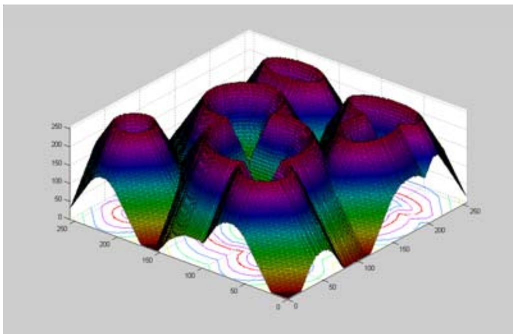
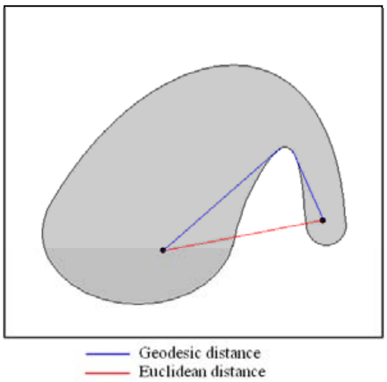


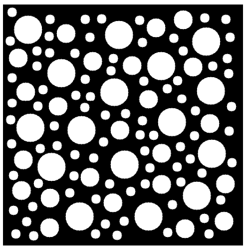


Geodesic distance

- Euclidean distance: shortest distance between two points.
- Geodesic distance: Length of the shortest path between two points along the object points.



Example - Sieving



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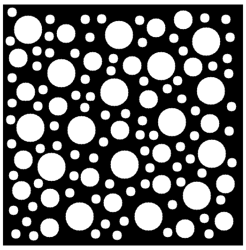

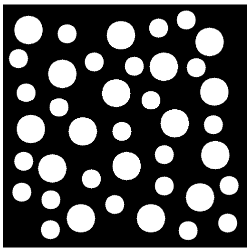

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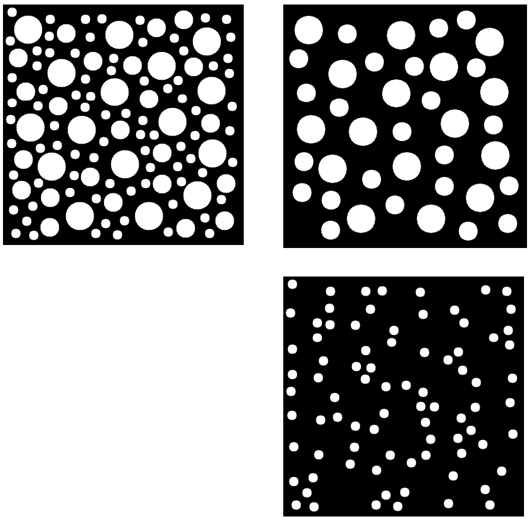

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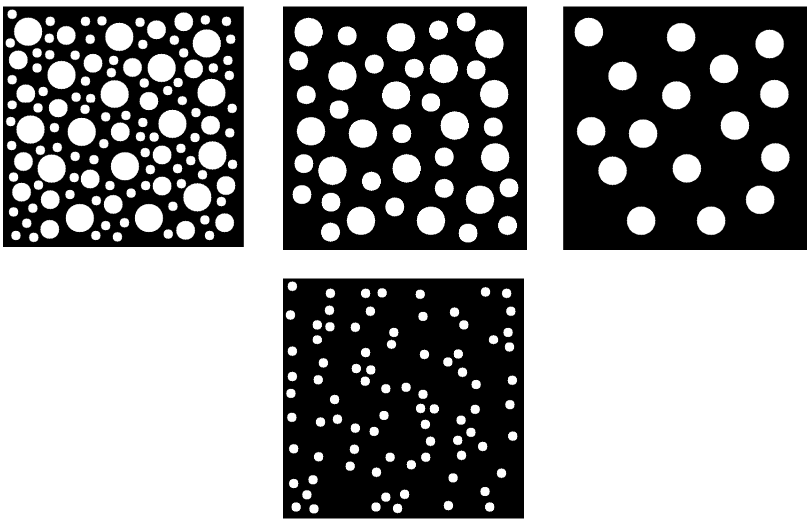

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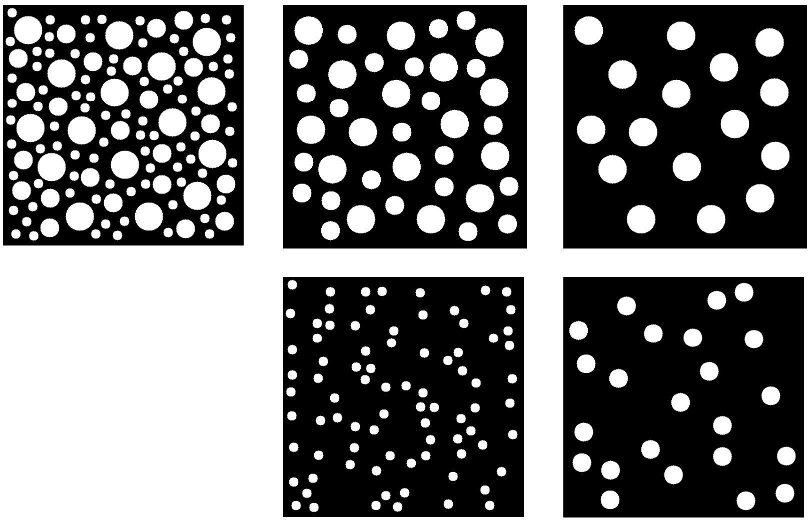


Scale or size parameter

Suppose B represents a compact point set of size one.

Then nB represents a set (or pattern or SE) of size n .

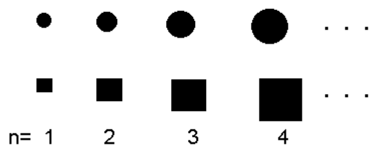
In continuous domain: $nB = \{nb \mid b \in B\}$

In discrete domain (if B is convex):

$$nB = \underbrace{B \oplus B \oplus \ldots \oplus B}_{n-1\ times}$$

for $n = 1,2,3,\ldots$ and if $n = 0, nB = \{0,0\}$

Homothetic sets



$$m < n \Rightarrow \quad 1.\ mB < nB$$

$$2.\ nB \circ mB = nB$$

Since opening and closing are increasing

$$\ldots \subseteq A \circ (n+1)B \subseteq A \circ nB \subseteq A \circ (n-1)B \subseteq \ldots \subseteq A$$

$$A \subseteq \ldots \subseteq A \bullet (n-1)B \subseteq A \bullet nB \subseteq A \bullet (n+1)B \subseteq \ldots$$

Properties of multi-scale processing

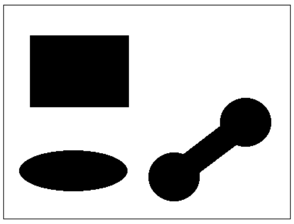
Multi-scale operation should satisfy the following properties:

1. **Causality**
   No new feature should be generated at the higher scale.

2. **Edge localization**
   Edge should not be drifted from its original position.

3. **Scale calibrated**
   Output should possess features of a single scale only.

Problem with traditional opening



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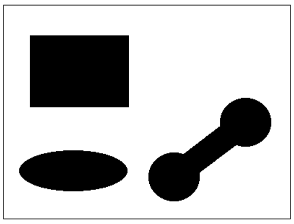

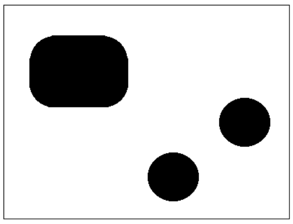

Both causality and edge localization problems occur.

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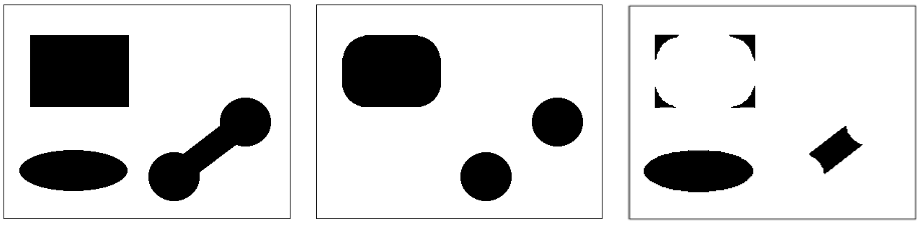


Both causality and edge localization problems occur.

Their difference is not scale calibrated too.

Reconstruction by dilation

Geodesic dilation of A w.r.t. B $(A \subset B)$ by X of size one:

$$\delta_x^1(A,B) = (A \oplus X) \bigcap B$$

Geodesic dilation of any size:

$$\delta_X^i(A,B) = (\delta_X^{(i-1)}(A,B) \oplus X) \bigcap B$$

There exists an n such that $\delta_X^n(A,B) = \delta_X^{(n+1)}(A,B)$ then

$$\delta^{(rec)}(A,B) = \delta^n(A,B)$$

In case of functions *f* and *g (f < g)*, reconstruction *R(f,g)* is done in a similar way, where ***intersection*** is replaced by ***infimum*** or ***minimum***.

Opening by reconstruction (ObR)

Geodesic dilation of A with respect to B by X of size one:

$$\delta_x^1(A,B) = (A \oplus X) \cap B$$

Geodesic dilation of any size:

$$\delta_X^i(A,B) = (\delta_X^{(i-1)}(A,B) \oplus X) \cap B$$

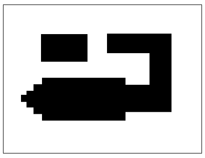
There exists a n such that $\delta_X^n(A,B) = \delta_X^{(n+1)}(A,B)$ then

$$\delta^{(rec)}(A,B) = \delta^n(A,B)$$

If A is the result of opening B by SE C, then we have ObR

$$B \breve{\circ} C = \delta^{(rec)}(B \circ C, B)$$

An example



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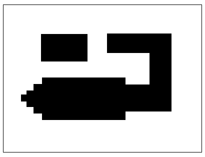

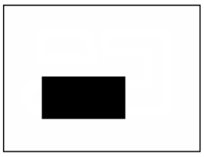

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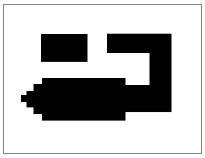

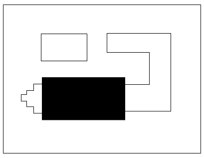

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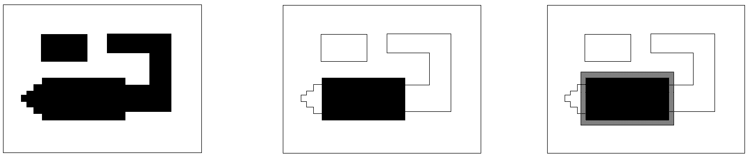

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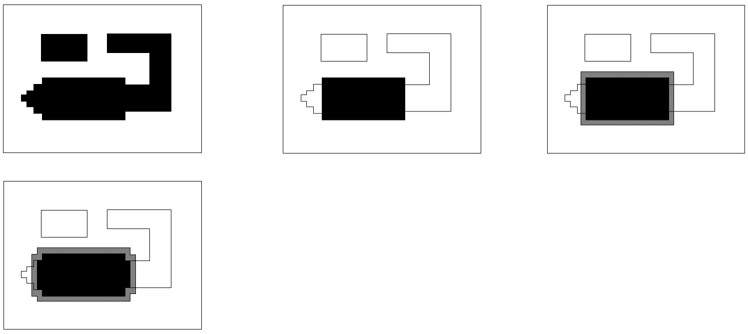

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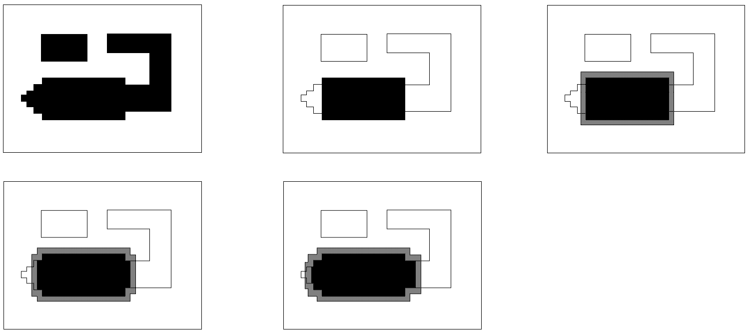

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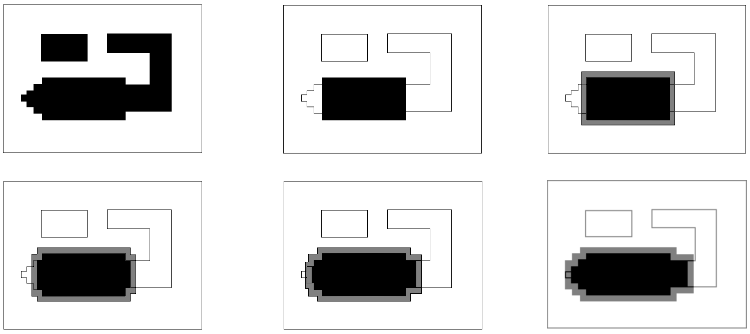

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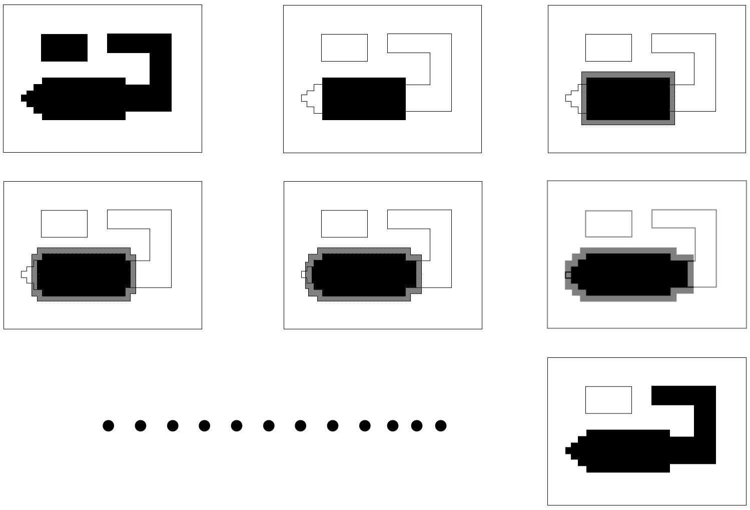
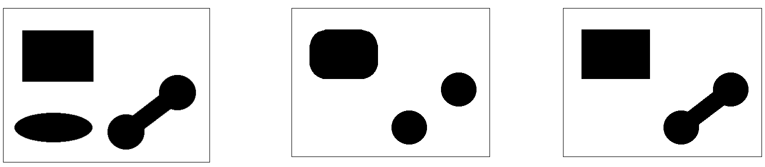


Result of ObR



Input Opening Opening by Recons.

No causality or edge localization problem occurs in case of Opening by reconstruction.

Influence zone (IZ)

- Let A be any set (marker) included in set X.
- Geodesic distance of any point x of X from A is the minimum of distance of x from any point a of A, i.e., $d(x, A) = \min_a \{d(x, a) \mid a \in A\}$
- If X contains two markers: A and B. IZ of A is a set of points x of X, which are closer to A than B, i.e.

$$IZ(A) = \{x \in X \mid d(x, A) < d(x, B)\}$$

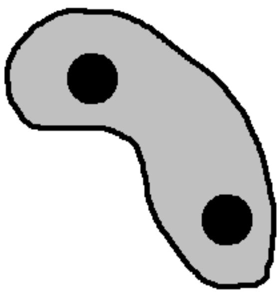
Skeleton by IZ (SKIZ)

- SKIZ is a line or boundary where distance from two markers are same, i.e.,

$$SKIZ(A) = \{x \in X\}$$

Such that $d(x, A) = d(x, B)$

- The concept is true for n number of markers.



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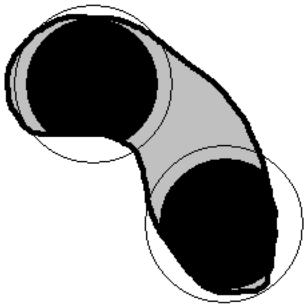

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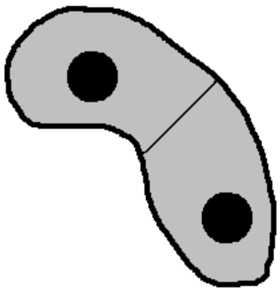


Reconstruction by erosion

Geodesic erosion of A w.r.t B $(A \supseteq B)$ by X of size one:

$$\varepsilon_x^1(A,B) = (A \Theta X) \bigcup B$$

Geodesic dilation of any size:

$$\varepsilon_X^i(A,B) = (\varepsilon_X^{(i-1)}(A,B) \Theta X) \bigcup B$$

There exists a n such that $\varepsilon_X^n(A,B) = \varepsilon_X^{(n+1)}(A,B)$ then

$$\varepsilon^{(rec)}(A,B) = \varepsilon^n(A,B)$$

In case of functions *f* and *g (f > g)*, reconstruction *R'(f,g)* is done similar way and ***union*** is replaced by ***maximum***.

Marker detection

- To find the minima of function *f(x)*, *f(x)+1* is reconstructed by erosion w.r.t. *f(x)*, i.e.,

  R'(f+1,f)

- Let us define a function

$$m(x) = R'(f(x)+1, f) - f(x)$$

- Now the minima or ***markers*** are defined as

$$M = \{M_i\}$$

where $M_i = \{x \mid m(x) = 1\}$

Marker detection

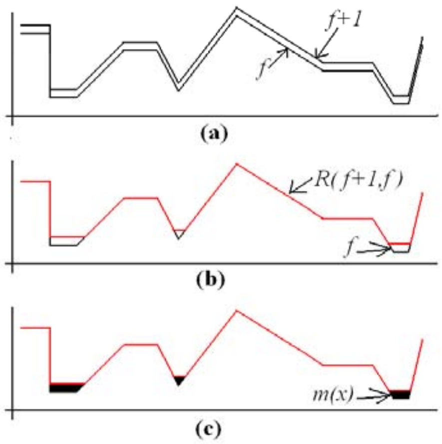
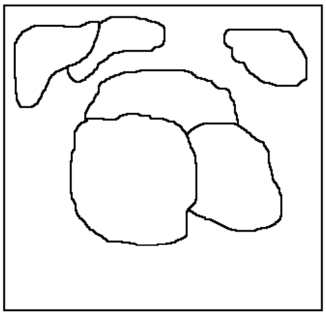


Image segmentation

- Input:
  - A graylevel image
- Output:
  - Partition of image domain based on consistent image features by say extracting boundary between homogeneous regions.

An example of segmentation

Original image



Desired output

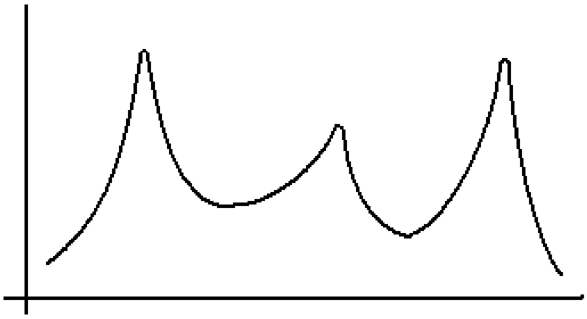
Watershed algorithm

Relevant terms: 1. Catchment basin

2. Watershed line

3. Markers

Watershed algorithm: 1-D



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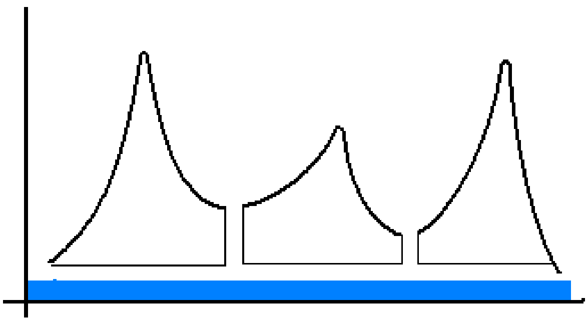
Watershed algorithm: 1-D



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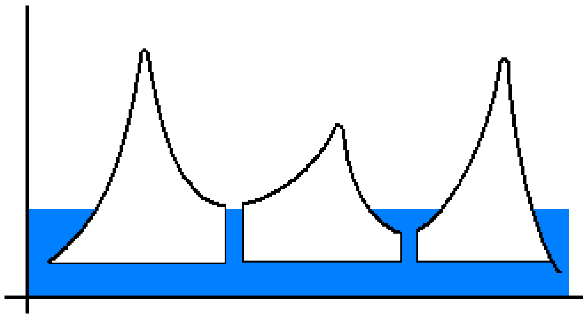
Watershed algorithm: 1-D



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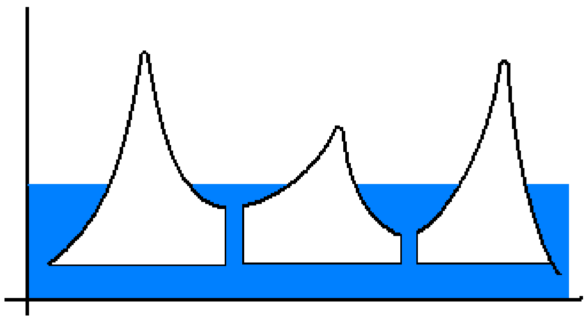
Watershed algorithm: 1-D



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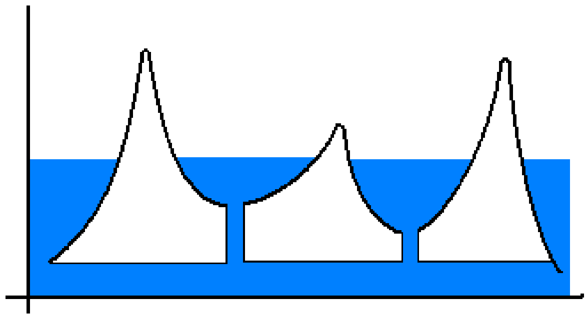
Watershed algorithm: 1-D



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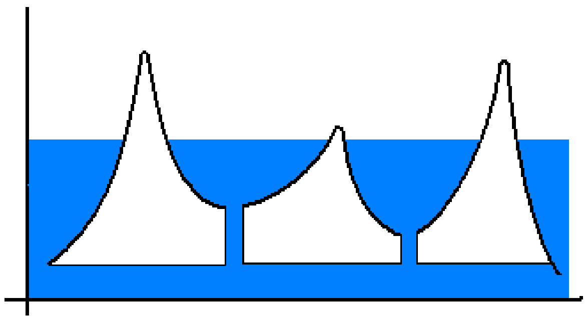
Watershed algorithm: 1-D



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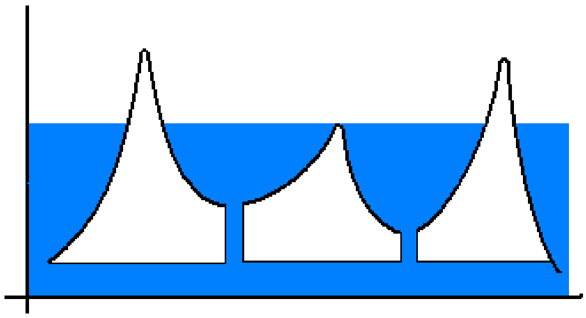
Watershed algorithm: 1-D



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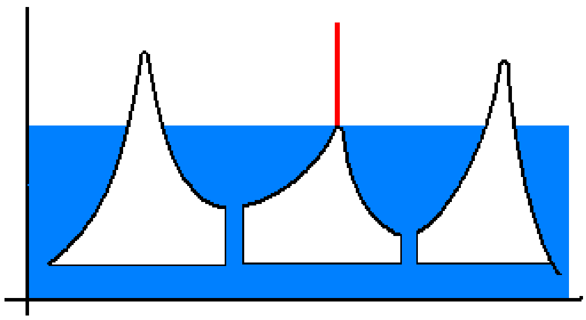
Watershed algorithm: 1-D



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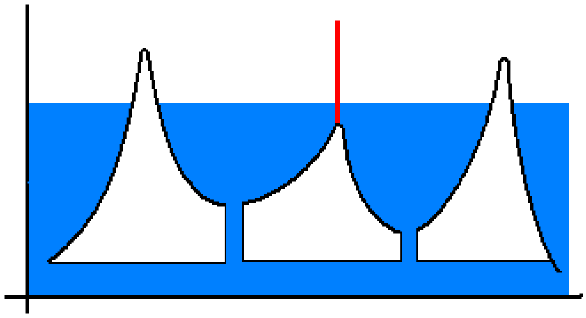
Watershed algorithm: 1-D



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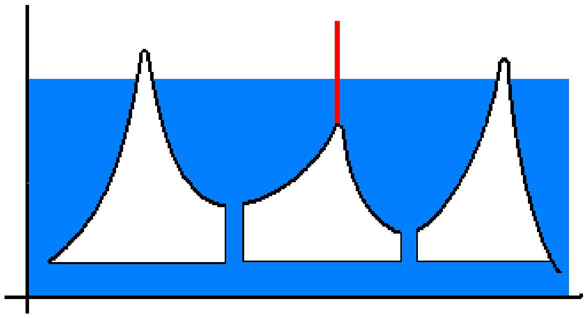
Watershed algorithm: 1-D



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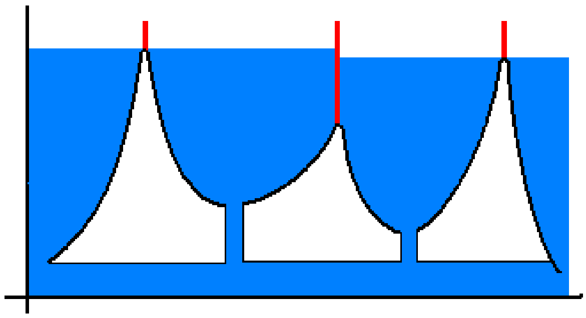
Watershed algorithm: 1-D



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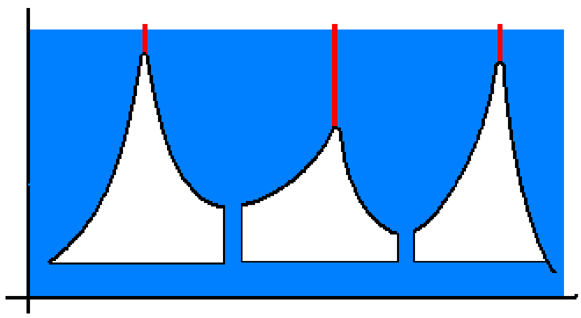
Watershed algorithm: 1-D



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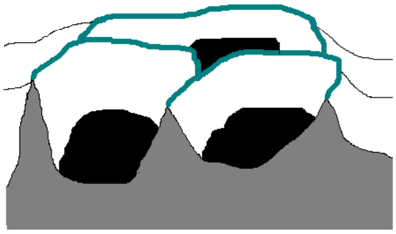
2-D case or image segmentation

Gradient image



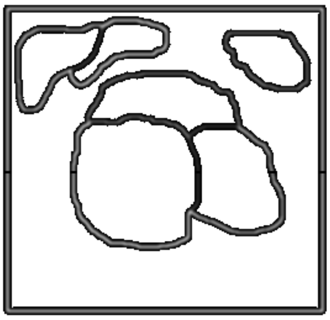
Green watershed lines are desired boundary between regions

Start with gradient image!

Gradient image: starting point

Blue marker: entry point of water from below

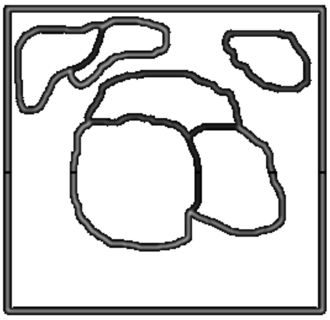
Start with gradient image!



Gradient image: starting point

Flooding with water …

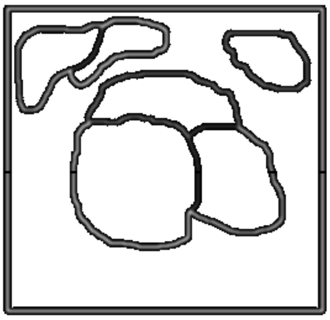
Start with gradient image!



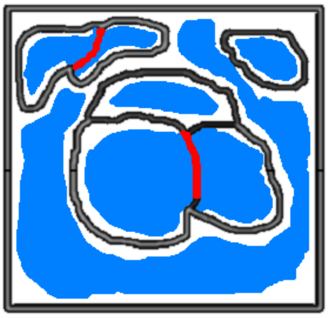
Gradient image: starting point

Forming dam(s) where water from two catchment basins meet.

Start with gradient image!

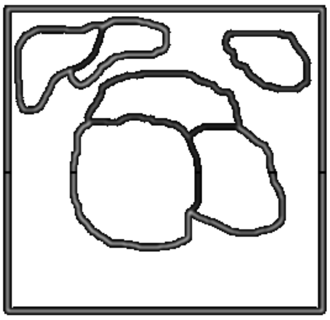


Gradient image: starting point

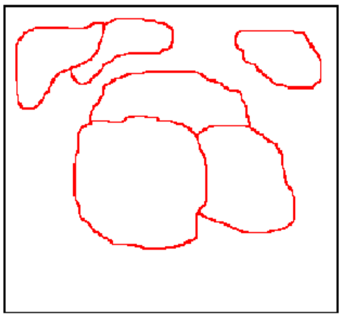


More flooding, more dams.

Final result

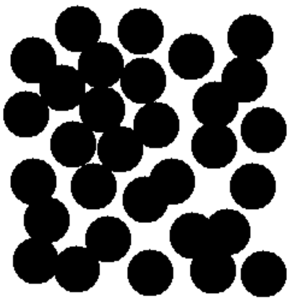


Gradient image: starting point

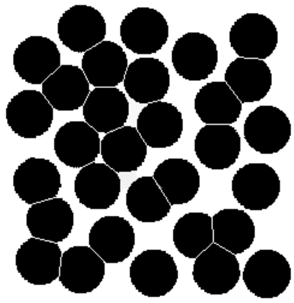


Output: red dams are boundary of the regions.

Segmenting binary objects

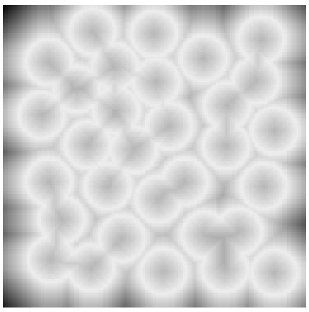


Original image

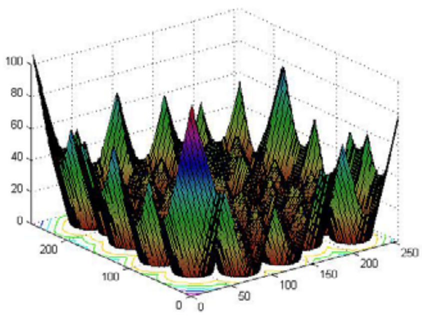


Desired output

Segmenting binary objects

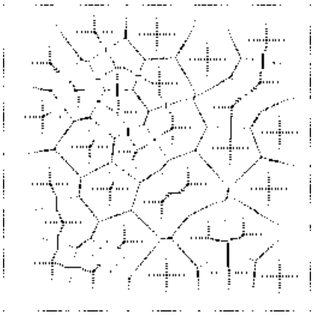


Distance transform of image

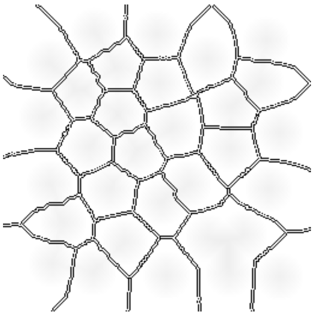


3D surface representation

Segmenting binary objects

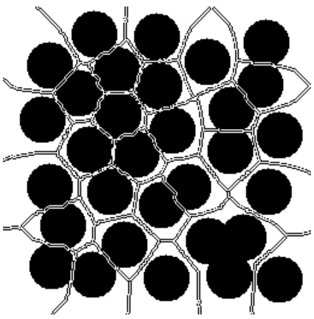


Markers of the image

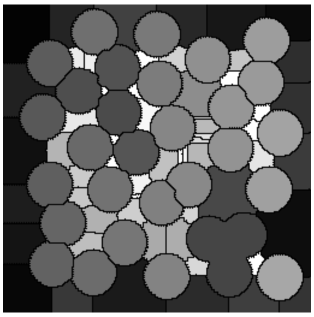


SKIZ of the objects

Segmenting binary objects

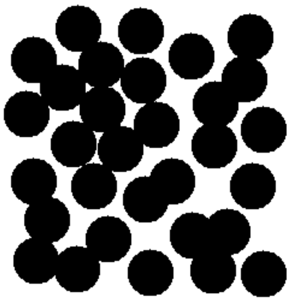


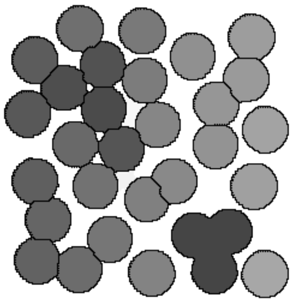
SKIZ superposed on image



Labeled isolated objects

Segmenting binary objects

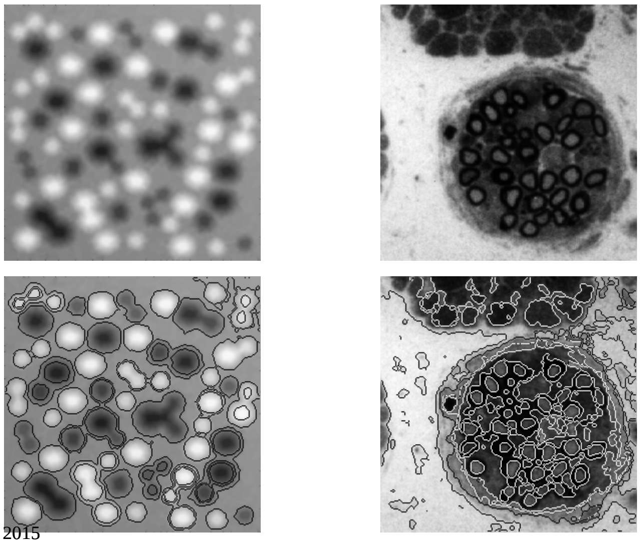




Original image used as mask

Final result

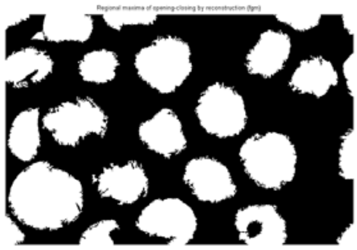
Result of segmentation



Watershed: Result

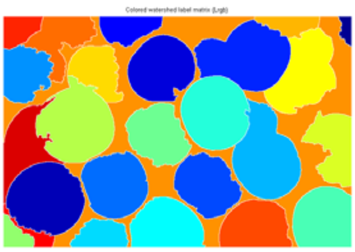
Input

Marker

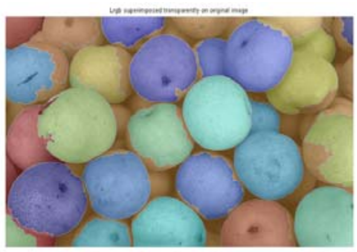


Watershed: Result

Segmented output



Superposed on input



Conclusion

- Watershed transformation does not need threshold value for edge detection.
- Watershed in its basic approach usually results in over-segmentation due to presence of many local minima.
- Over-segmentation may be controlled by smoothing as well as marker selection.
- Computationally expensive unless special data structure is used.

References

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- Dougherty, E.R. and Giardina, C.R., Morphological Methods in Image and Signal Processing, Prentice Hall, Englewood Cliffs, New Jersey, 1988.
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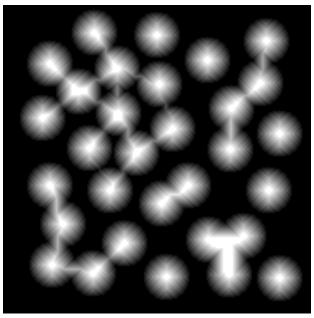
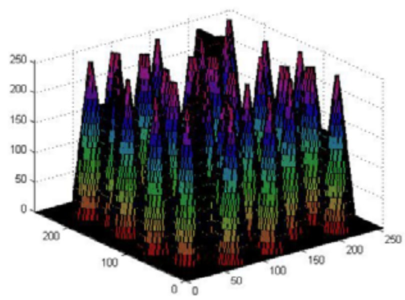
- Beucher, S. and Meyer, F., The morphological approach to segmentation: the watershed transformation. In *Mathematical Morphology in Image Processing* (Ed. E.R. Dougherty), 1993.

Thank You.

Segmenting binary objects

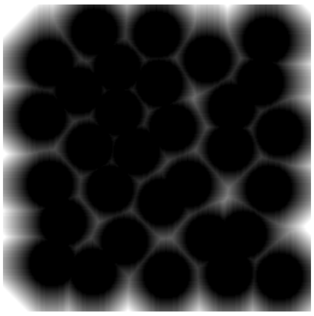


Distance transform of objects

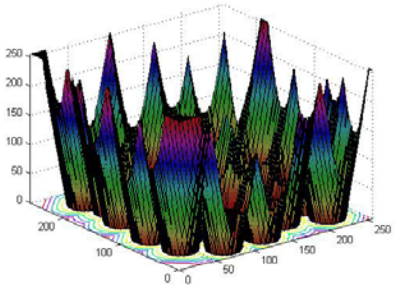


3D surface representation

Segmenting binary objects



Distance transform
of background



3D surface representation