

Poisson process approximation under stabilization and Palm coupling.



D. Yogeshwaran
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November 17, 2022

Probability Victoria Seminar

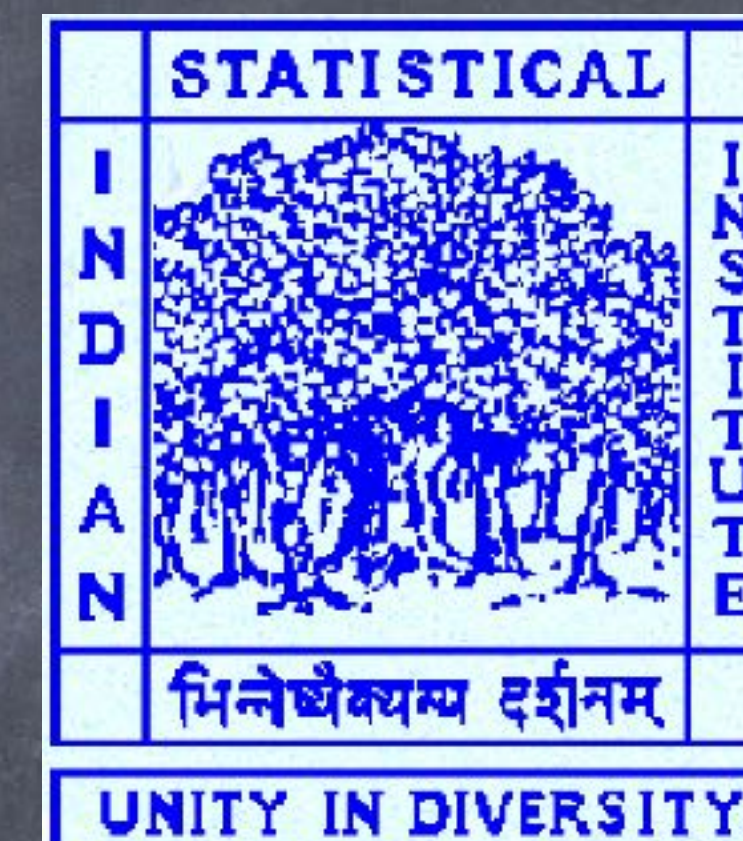
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Accepted in Ann. Henri Lebesgue.

Omer Bobrowski
Technion, Israel &
Queen Mary's Univ. London



Matthias Schulte
TU Hamburg, Germany



PART I : k-nearest neighbour distances

PART II : Abstract Poisson process approximation
under Stabilisation

PART III : Abstract Poisson process approximation
under Palm coupling.

PART IV : Generator Approach to Stein's Method

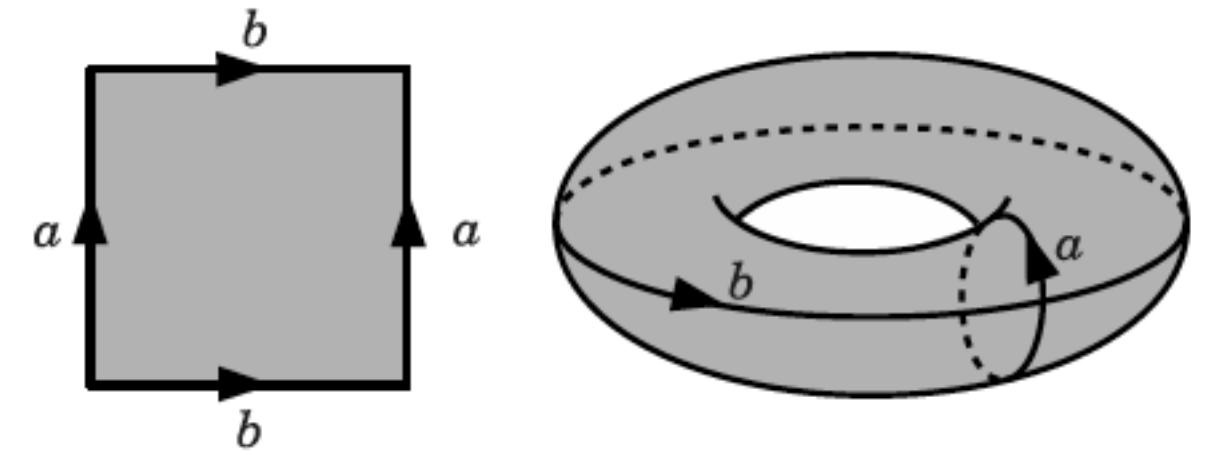
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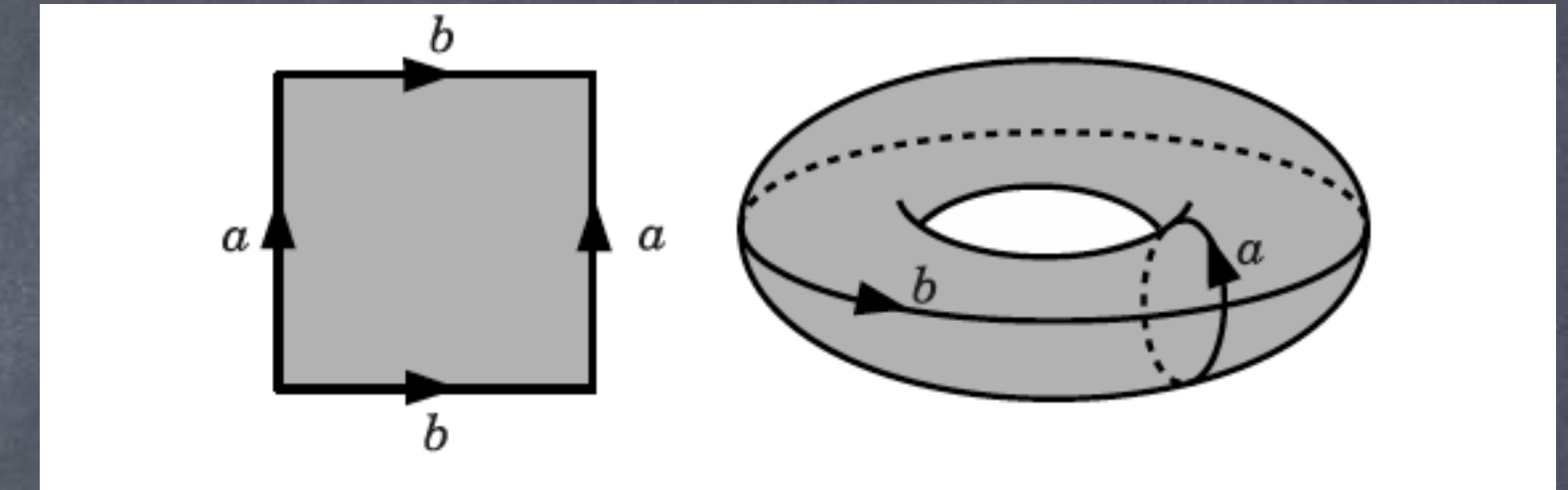
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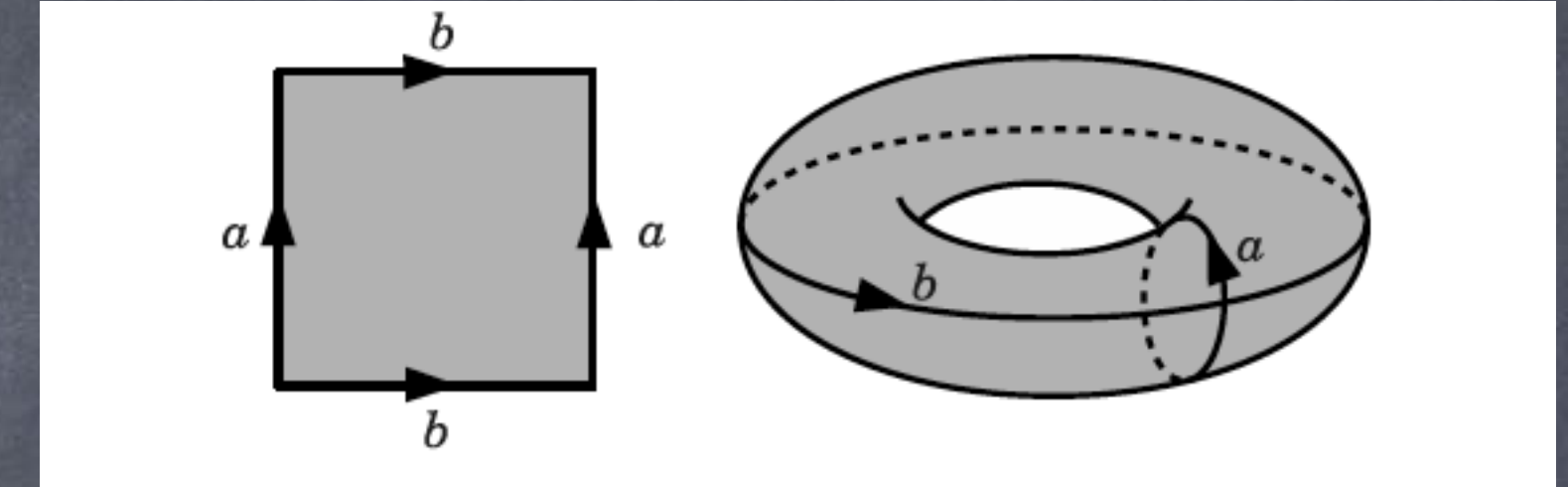
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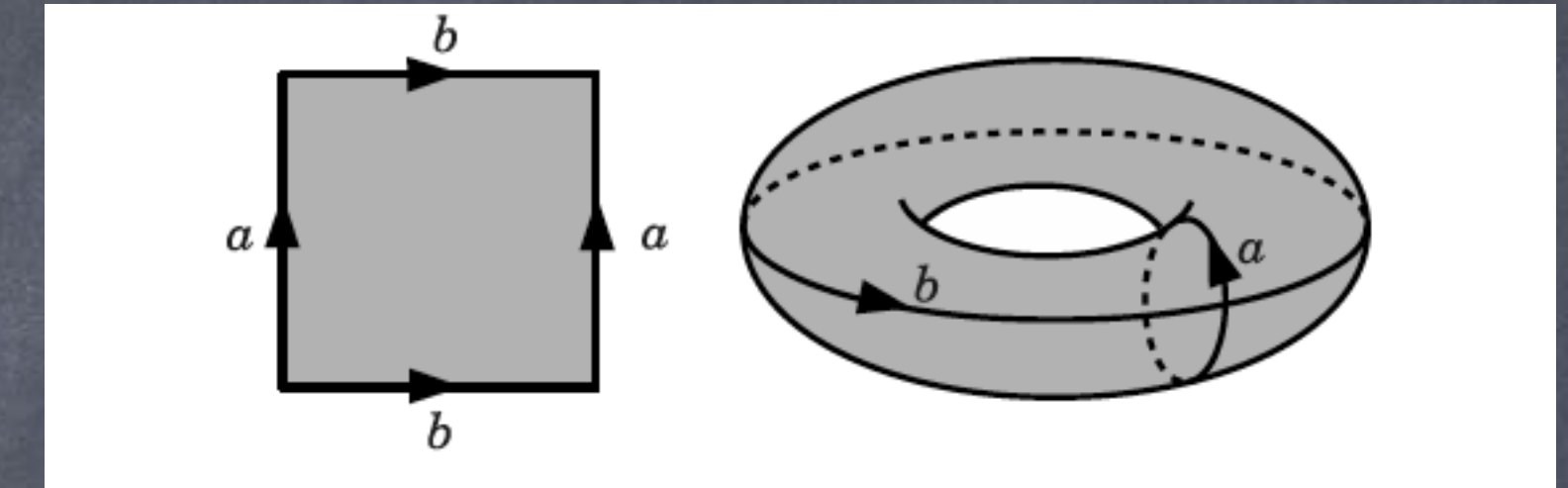
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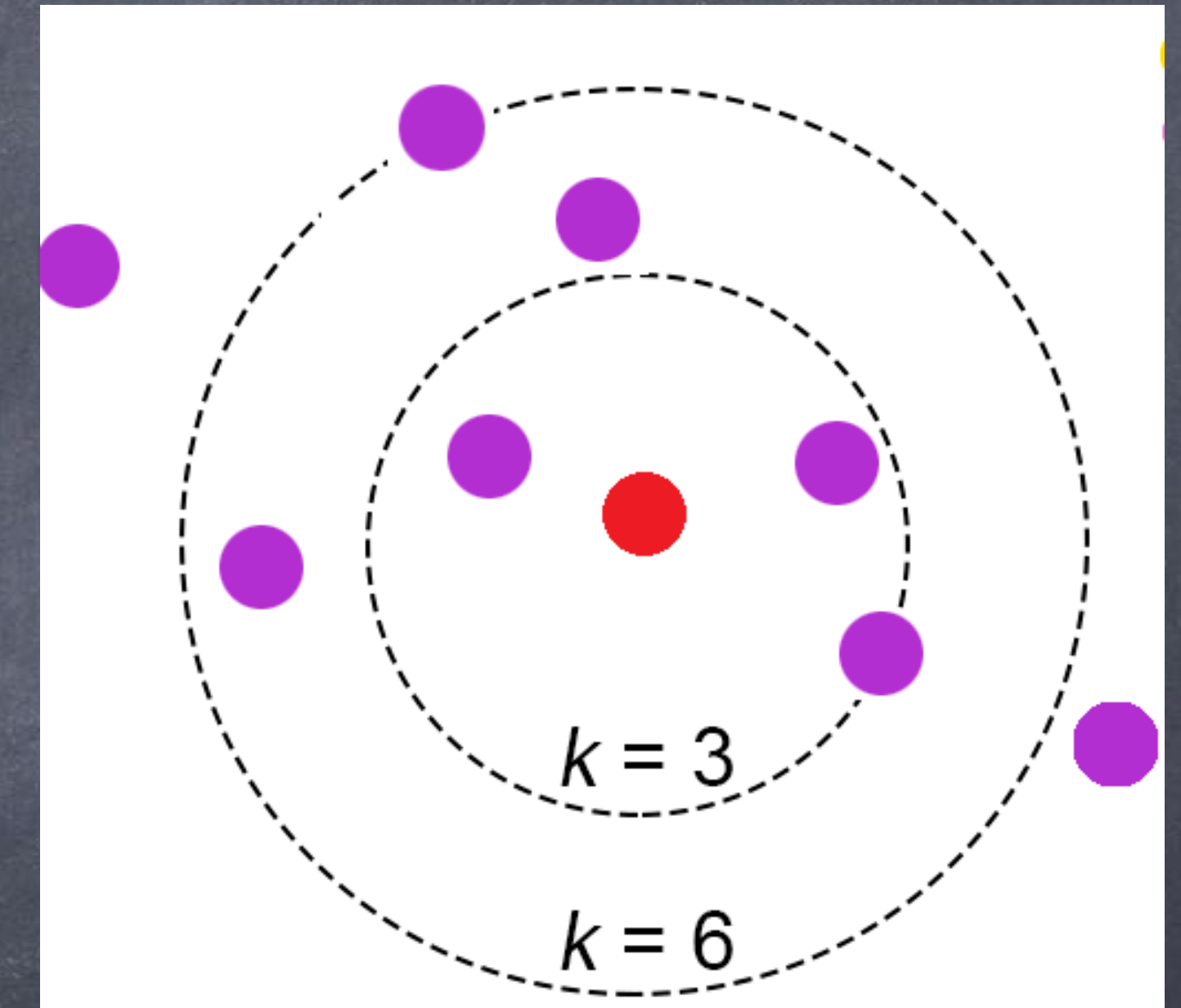
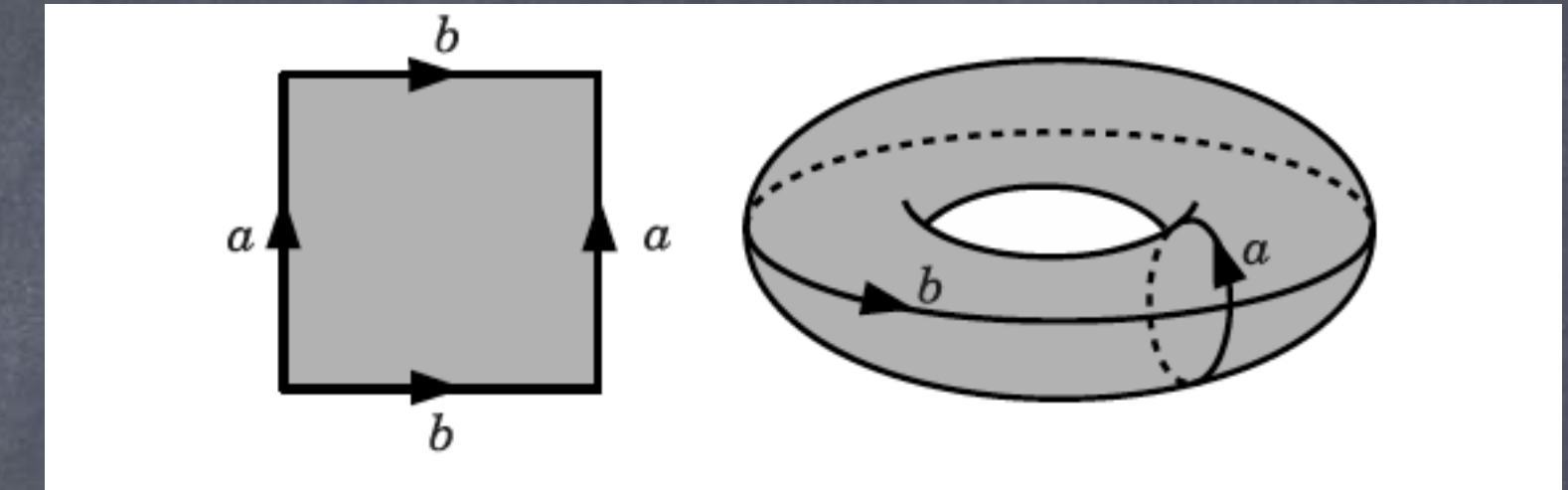
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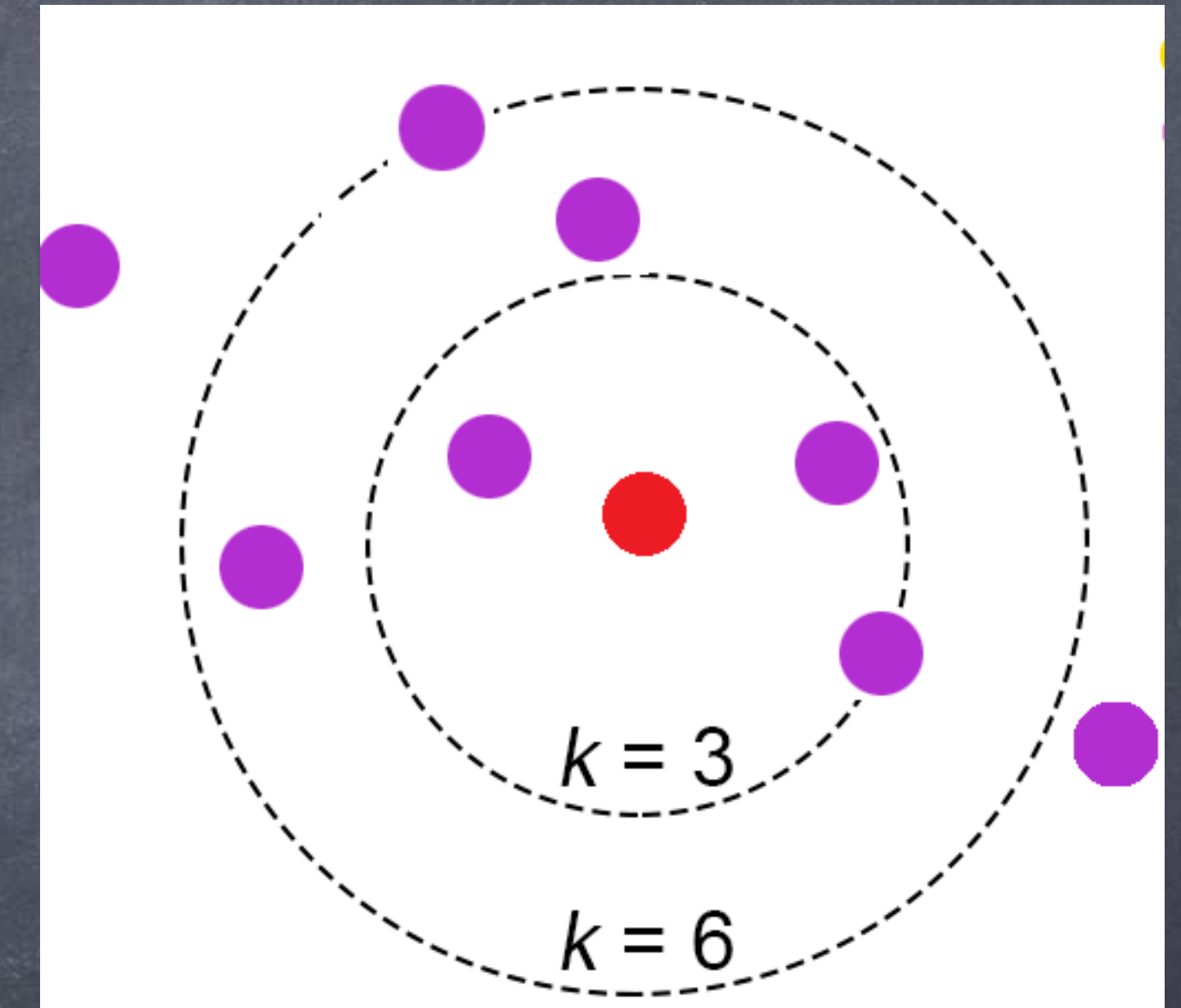
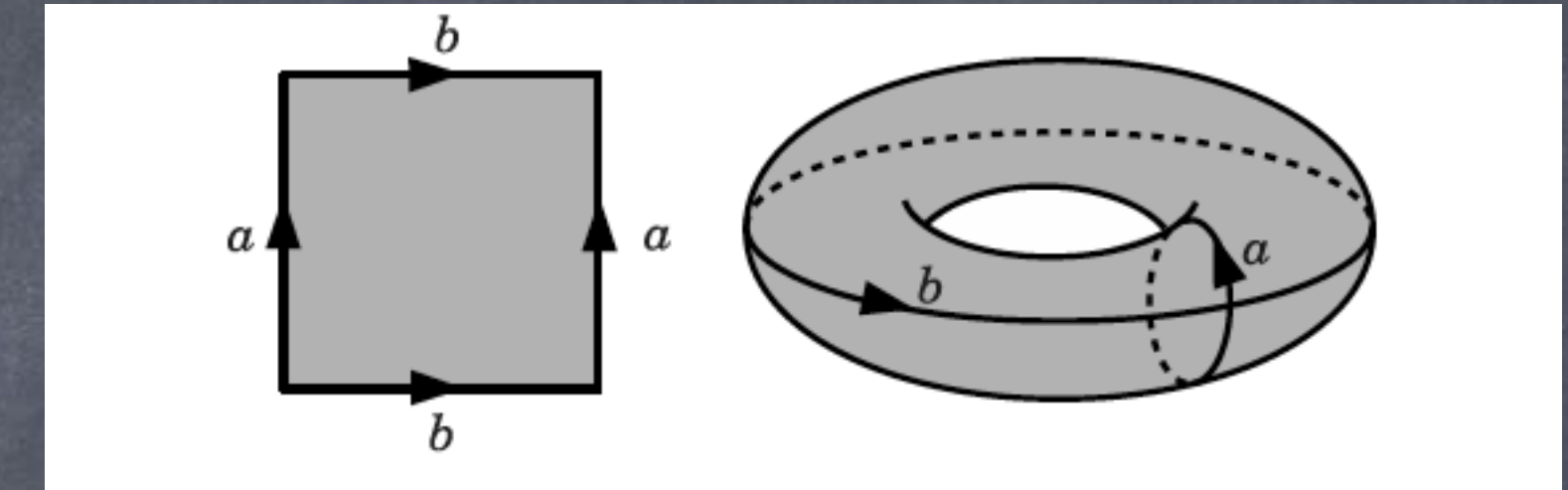
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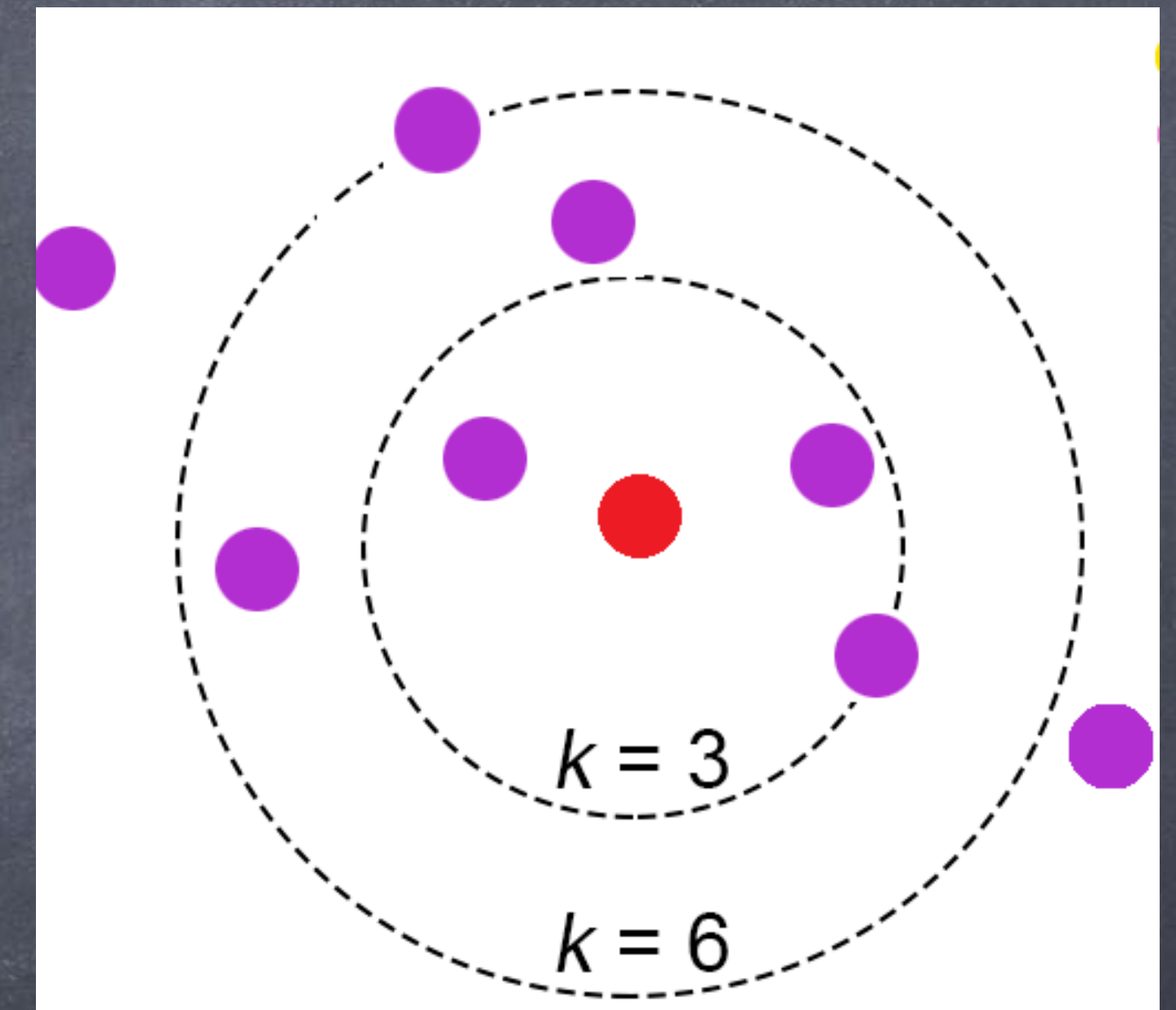
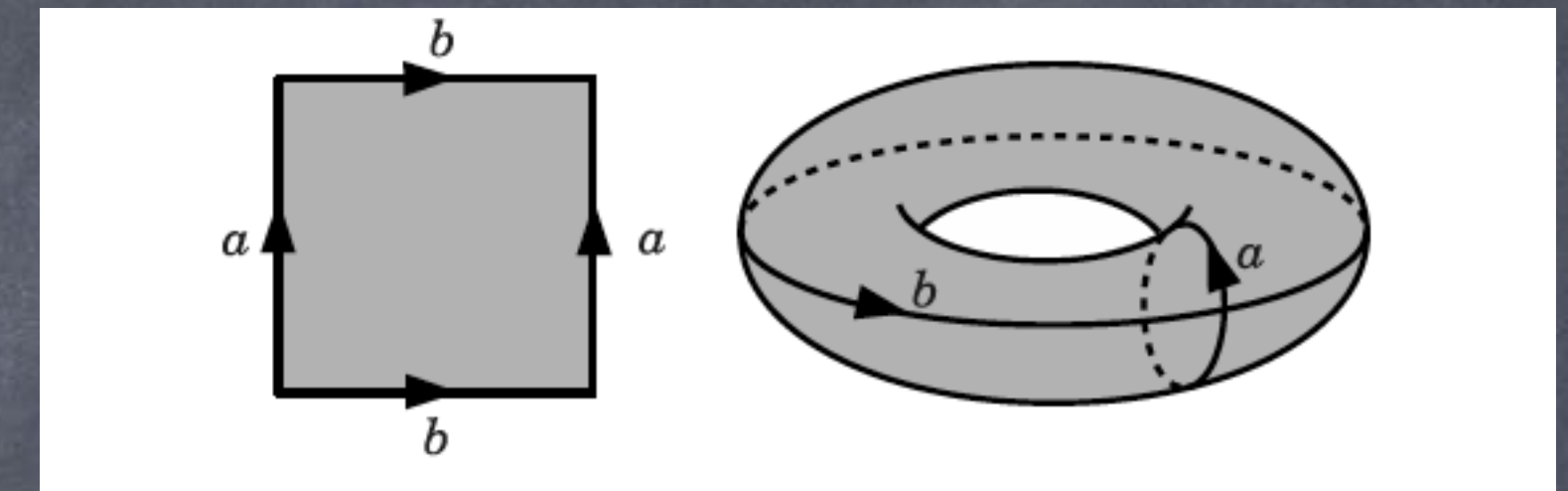
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Applications:

- Computational geometry
- k-nearest neighbour methods in statistics (see [Penrose, 2003](#))
- Related to minimal spanning tree on η_n (see [Penrose, 1997](#))
- Simple example of a quasi-local statistic (later in the talk)

Largest k-nearest neighbour distances

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$\xrightarrow{d}$ weak convergence of Radon counting measures under vague topology.

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- Our result holds for Binomial process – $\{X_1, \dots, X_n\}$.

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$$\xi_n^a[\eta_n] := \{(X_i, n\theta_d R_i^d - a_n) : n\theta_d R_i^d - a_n \geq a, 1 \leq i \leq N_n\} \subset \mathbb{T}^d \times [a, \infty) \quad a \in \mathbb{R}.$$

THEOREM : (Bobrowski, Schulte, Y. 2021) ζ^a – PP on $\mathbb{T}^d \times [a, \infty)$ with intensity $e^{-t} dt dx$.

$$d_{KR}(\xi_n^a, \zeta^a) \leq C \frac{\log \log n}{\log n}, \quad n \geq 3. \quad d_{KR} - \text{Kantorovich-Rubinstein distance.}$$

- $d_V \leq d_{KR}$ and so this gives results of Otto (2020) and Penrose (1997).
- Penrose (1997) is only for homogeneous Poisson process (i.e., X_i uniform) while Otto (2020) and our results hold also for inhomogeneous Poisson process (i.e., X_i non-uniform).
- Our result holds for Binomial process – $\{X_1, \dots, X_n\}$.
- Penrose (2018) implies similar bounds for $d_{Wa}(|\xi_n^a|, |\zeta^a|)$

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$c_n - a_n \rightarrow -\infty$. LDP (weak topology) for $\frac{1}{b_n} \sum_{X_i \in \eta_n} \delta_{(X_i, n\theta_d R_i^d - c_n)}$ with rate b_n and explicit rate function.

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- $c_n - a_n \rightarrow \infty$, $c_n = o(n)$. LDP ($\mathcal{M}_0$ -topology) for $\sum_{X_i \in \eta_n} \delta_{(X_i, n\theta_d R_i^d - c_n)}$ with rate b_n and explicit rate function.

PART II : Abstract Poisson process approximation under Stabilisation

“The proper business of probabilists is calculating probabilities. . . . A limit theorem is an assertion of the form: ‘the error in a certain approximation tends to 0 as (say $N \rightarrow \infty$). Call such limit theorem naive if there is no explicit error bound.”

– David J. Aldous,

Probability Approximations via the Poisson Clumping Heuristic

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- **Our theorem**: $\xi_n^a[\eta_n] = \lim_{b \rightarrow \infty} \xi_n^{a,b}[\eta_n]$; induced by **quasi-local** statistics.

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$$\text{"Easy term"} = \int \mathbf{1}[S(x) \cap S(z) \neq \emptyset] \mathbb{E}[g(x, \eta + x)] \mathbb{E}[g(z, \eta + z)] K^2(dx dz)$$

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- **Another extension by Otto (2021).**

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- For ex., LDP for Morse critical points above the "vanishing threshold" – ongoing with Takashi Owada and Zifu Wei.

PART III : Abstract Poisson process approximation under Palm coupling.

A MORE GENERAL APPROXIMATION QUESTION

- Ingredients

ζ – Poisson process on $\mathbb{X}$, lcsch space. Finite intensity measure $\mathbf{K}$

ξ – point process on $\mathbb{X}$, lcsch space. Finite intensity measure $\mathbf{L}$ ($= \mathbf{K}$, WLOG)

- K-R distance** – $d_{KR}(\xi, \zeta) := \sup\{ |\mathbb{E}[h(\xi)] - \mathbb{E}[h(\zeta)]| : h \text{ Lipschitz w.r.t } d_{TV} \}$

HOW TO BOUND $d_{KR}(\xi, \zeta)$?

PALM THEORY AND MECKE THEOREM

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Formally, for all $f \geq 0$, $\mathbb{E} \sum_{x \in \xi} f(x, \xi) = \int_{\mathbb{X}} \mathbb{E}[f(x, \xi_x^! + \delta_x)] \mathbf{K}(dx)$.

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Mecke Theorem: If $\xi_x^! \stackrel{d}{=} \xi \forall \text{ a.e. } x \in \mathbb{X}$, then ξ is a Poisson process.

PALM THEORY AND MECKE THEOREM

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Reduced Palm Version – $\xi_x^!$: $\xi_x^! \stackrel{d}{=} \xi - \delta_x \mid x \in \xi$

Formally, for all $f \geq 0$, $\mathbb{E} \sum_{x \in \xi} f(x, \xi) = \int_{\mathbb{X}} \mathbb{E}[f(x, \xi_x^! + \delta_x)] \mathbf{K}(dx)$. **Campbell–Mecke–Little Formula.**

ζ – Poisson process on $\mathbb{X}$, then $\zeta_x^! \stackrel{d}{=} \zeta \forall x \in \mathbb{X}$.

Mecke Theorem: If $\xi_x^! \stackrel{d}{=} \xi \forall \text{ a.e. } x \in \mathbb{X}$, then ξ is a Poisson process.

Idea for approximation: If $\xi_x^! \approx \xi \forall \text{ a.e. } x \in \mathbb{X}$, then $\xi \approx \zeta$, a Poisson process !

APPROXIMATION VIA PALM COUPLING

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- TV bounds in Barbour and Brown (1992) without factor of 2. Used by Otto (2020) for quasi-local statistics.

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- Alternative coupling bounds are provided. For eg., $\xi[\eta + \delta_x] - \delta_{f(x)} \mid \{g(x, \eta + \delta_x) = 1\}.$

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- **Total count :** Extension of bound for Poisson approximation of $\xi(\mathbb{Y})$ by Penrose (2018).

PART IV : *Generator Approach to Stein's Method*

GENERAL POISSON PROCESS APPROXIMATION

Ingredients

ζ – Poisson process on $\mathbb{X}$, lcsch space. Finite intensity measure $\mathbf{K}$

ξ – point process on $\mathbb{X}$, lcsch space. Finite intensity measure $\mathbf{L}$ ($= \mathbf{K}$, WLOG)

K-R distance – $d_{KR}(\xi, \zeta) := \sup\{ |\mathbb{E}[h(\xi)] - \mathbb{E}[h(\zeta)]| : h \text{ Lipschitz w.r.t } d_{TV} \}$

HOW TO DERIVE BOUNDS FOR $d_{KR}(\xi, \zeta)$?

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$$\mathcal{L}h(\mu) := \int_{\mathbb{X}} D_x h(\mu) \mathbf{K}(\mathrm{d}x) - \int_{\mathbb{X}} D_x h(\mu - \delta_x) \mu(\mathrm{d}x)$$

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Stein Equation:

$$h(\mu) - \mathbb{E}[h(\zeta)] = \int_0^\infty \mathcal{L}P_s h(\mu) ds$$

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$$\bullet \quad d_{KR}(\xi, \zeta) := \sup\{ |\mathbb{E}[h(\xi)] - \mathbb{E}[h(\zeta)]| : h \text{ Lip.} \} \leq \sup\{ \int_0^\infty |\mathbb{E}[\mathcal{L}P_s h(\xi)]| \mathrm{d}s : h \text{ Lip.} \}$$

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- **Our Proof** : Derive different bounds for $\mathbb{E}[\mathcal{L}h(\xi)]$ Lipschitz h via Palm coupling and local dependence i.e., stabilization.

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