Random Graphs

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- $d(f,g) =$

$$\sup_{\substack{h,k;\|h\|_\infty \leq 1 \\ \|k\|_\infty \leq 1}} \int_{[0,1]^2} [f(x,y) - g(x,y)]h(x)k(y)dxdy$$

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- $h(Sf) = h(f)$. LSC implies that if $f \simeq g$ then $g_n = S_n f \to g$
- $$d(S_n f, g) \to 0$$
- $$d(f, S_n^{-1} g) \to 0$$

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- Let us define a collection of functions $\{f(x,y)\}$.
- For each ℓ the unit interval is divided into $\ell + 1$ subintervals $(J_0, \ldots, J_{\ell+1})$,
- ℓ of them are of equal length and the last one is of length at most ϵ.

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- A finite collection of discs $\{D_j\}$ of radius ϵ in the cut topology will cover this.

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- The union of orbits of this finite collection of discs contains the functions that correspond to all graphs.
- Any function f can be approximated by functions that correspond to graphs with enough vertices.
- Replace f by g that is piecewise constant on a finite grid. Can do it L_1 and so in cutmetric.

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- Therefore the space $\mathbf{Y}$ can be covered by a finite number of discs of size 3ϵ for any $\epsilon > 0$.
- $\mathbf{Y}$ is totally bounded.
- The space $\mathbf{Y}$ is complete. $\tilde{d}_{\square}(f_n, f_m) \to 0$.

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- Use the completeness of $d_\square$

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- After all, any cut neighborhood is contained in a weak neighborhood, and the upper bound estimate holds.
- But to prove in $\mathbf{Y}$ we need to get the same estimate on the orbit of the disc by the group S.

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- $$P_{n,p}[D(\tilde{f},\eta)] \leq n! P_{n,p}[D(\tilde{f},\eta) \cap K^\epsilon]$$
- Finite number of discs $D_j^{2\epsilon_j}$. Take one.

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- LDP bound for $D(g,\epsilon)$ is enough.

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- If $\frac{p^3}{6} \leq t \leq s \leq 1$, then with $(1-\delta)^3 = \frac{t}{s}$

$$I_p(t) \leq [\frac{t}{s}]^{\frac{1}{3}} I_p(s)$$

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- $I_p(t) \leq h_p(f_\delta) \leq (1-\delta)h_p(f) = [\frac{t}{s}]^{\frac{1}{3}}$ by convexity.

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- If $h_p(t) = \hat{h}_p(t)$ i.e they touch at some point t then
- $f(x, y) = (6t)^{\frac{1}{3}}$ is the optimizer for that (p, t) and the graph looks like an Erdös-Renyi graph with a new $p = (6t)^{\frac{1}{3}}$.

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- Let c_t be the function identically equal to $(6t)^{\frac{1}{3}}$

- $at - h_p(c_t) = \int [\frac{a[c_t(x,y)]^3}{6} - h_p(c_t(x,y))]dxdy$

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- Hölder inequality.

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- $c(t) \simeq \frac{1}{2}(6t)^{\frac{1}{3}} \log \frac{1}{p}$
- $c'(t) < c(t)$

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$$\sup_g [F(g) - h_p(g)]$$

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- Finally the correct constant was worked out. (with additional conditions.)