Metastability for the dilute Curie–Weiss model with Glauber dynamics

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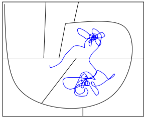
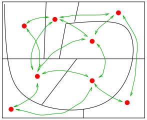
TU Delft — Delft University of Technology

What is metastability?

Metastability is a phenomenon where a system, under the influence of a stochastic dynamics, moves between different regions of its state space on different time scales.





Fast time scale:
quasi-equilibrium within single subregion

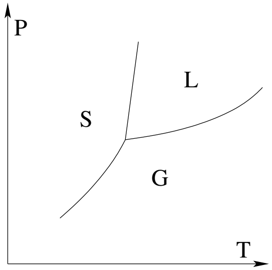
Slow time scale:
transitions between different subregions

Monographs:

- Olivieri and Vares 2005
- Bovier and den Hollander 2015

Metastability in Statistical Physics

Metastability is the dynamical manifestation of a first-order phase transition.



When the parameters of a system are changed rapidly in such a way that they cross the coexistence line, we see that the system will persist for long time in a metastable state before transiting (rapidly) to the new stable state under some random fluctuations.

An example is condensation:
metastable state: gas (supersaturated gas),
stable state: liquid

...But most systems of interest in statistical physics have many degrees of freedom. Strategy: mapping to a low-dimensional state space, or coarse-graining. Look first at a model where coarse-graining works perfectly.

The Curie–Weiss model

The Curie–Weiss model is the simplest model for a ferromagnet.
Spin model on $[N] = \{1, 2, \ldots, N\}$.
Configuration space $\mathcal{S}_N = \{-1, +1\}^N$
Configuration $\sigma = (\sigma_i)_{i\in[N]} \in \mathcal{S}_N$, $\sigma_i \in \{-1, +1\}$
Hamiltonian in the standard **Curie–Weiss model** (CW)

$$H_N^{\mathsf{CW}}(\sigma) = -\frac{1}{N} \sum_{1\leq i<j\leq N} \sigma_i\sigma_j - h \sum_{i\in[N]} \sigma_i$$

$h > 0$ constant magnetic field.
$\Longrightarrow$ Mean-field model

Magnetization in the Curie-Weiss model

Note that "mean-field" means that $H_N(\sigma)$ depends on σ only through the empirical magnetization

$$m_N(\sigma) = \frac{1}{N}\sum_{i\in[N]}\sigma_i, \qquad m_N \in \Gamma_N = \left\{-1, -1+\tfrac{2}{N}, ..., 1-\tfrac{2}{N}, 1\right\}$$

$$\begin{aligned} H_N^{\mathsf{CW}}(\sigma) &= -\frac{1}{2N}\sum_{i,j=1}^{N}\sigma_i\sigma_j - h\sum_{i=1}^{N}\sigma_i = -\frac{1}{2N}\left(\sum_{i=1}^{N}\sigma_i\right)^2 - h\sum_{i=1}^{N}\sigma_i, \\ &= -N\left(\frac{1}{2}m_N(\sigma)^2 + hm_N(\sigma)\right) = NE(m_N(\sigma)) \end{aligned}$$

Equilibrium measure and free energy

At equilibrium we define the Gibbs measure, $\sigma \in \mathcal{S}_N$,

$$\mu^{\mathsf{CW}}_{N,\beta}(\sigma) = \frac{\mathrm{e}^{-\beta H^{\mathsf{CW}}_N(\sigma)}}{Z^{\mathsf{CW}}_{N,\beta}} \qquad \text{with} \qquad Z^{\mathsf{CW}}_{N,\beta} = \sum_{\sigma \in \mathcal{S}_N} \mathrm{e}^{-\beta H^{\mathsf{CW}}_N(\sigma)}$$

were $\beta \in (0,\infty)$ is the inverse temperature and $Z^{\mathsf{CW}}_{N,\beta}$ the partition function.
Mesoscopic measure on Γ_N:

$$\mathcal{Q}^{\mathsf{CW}}_{N,\beta}(m) = \mu^{\mathsf{CW}}_{N,\beta} \circ m_N^{-1}(m) = \frac{\mathrm{e}^{-\beta N F_{N,\beta}(m)}}{Z^{\mathsf{CW}}_{N,\beta}}$$

where $F_{N,\beta}$ is the free energy

$$F_{N,\beta}(m) = E(m) + \beta^{-1} I_N(m)$$

and I_N is the entropy

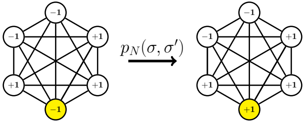
$$I_N(m) = -\frac{1}{N} \log \binom{N}{\frac{1+m}{2}N}$$

$\mathcal{S}_N[m] := m_N^{-1}(m)$

The Glauber dynamics

We consider a discrete time **Glauber dynamics** on $\mathcal{S}_N$ with $\beta > 0$ inverse temperature, i.e. a single spin flip with probability

$$p_N(\sigma, \sigma') = \frac{1}{N} \exp(-\beta [H_N^{\mathsf{CW}}(\sigma') - H_N^{\mathsf{CW}}(\sigma)]_+)$$



The **equilibrium** Gibbs measure $\mu_N^{\mathsf{CW}}(\sigma)$ is the **invariant** and **reversible** measure.

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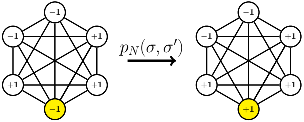

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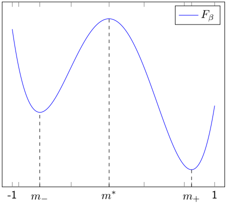


- $\lim_{N\to\infty} F_{N,\beta,h}(m) = F_{\beta,h}(m)$
- $\{m_-, m^*, m_+\}$: critical points of $F_{\beta,h}$ solve $m = \tanh\left(\beta\left[m+h\right]\right)$

- Hitting time of A

$$\tau_A = \inf\{t > 0 : \sigma_t \in A\}.$$

- $\mathbb{E}_\sigma$ is the expectation w.r.t. the Markov process for the CW model with Glauber dynamics starting in σ.

Metastability for the Curie-Weiss model

Theorem (Metastability for CW)

For $\beta \in (1,\infty)$, $h \in [0, \bar{h}_c(\beta))$ and uniformly in $\sigma \in \mathcal{S}_N[m_-]$, as $N \to \infty$,

- **Mean metastable exit time**

$$\mathbb{E}_\sigma\left[\tau_{\mathcal{S}_N[m_+]}\right] = \sqrt{\frac{1-m^{*2}}{1-m_-^2}}\,\frac{\mathrm{e}^{\beta N[F_{\beta,h}(m^*)-F_{\beta,h}(m_-)]}}{\sqrt{F''_{\beta,h}(m_-)[-F''_{\beta,h}(m^*)]}}\,\frac{\pi N[1+o(1)]}{\beta(1-m^*)}.$$

- **Exponential law**

$$\mathbb{P}_\sigma\left(\tau_{\mathcal{S}_N[m_+]} > t\,\mathbb{E}_\sigma\left[\tau_{\mathcal{S}_N[m_+]}\right]\right) = [1+o(1)]\,\mathrm{e}^{-t}, \qquad t \geq 0.$$

The randomly dilute Curie–Weiss model

The RDCW model is a classical model of a disordered ferromagnet.
$[N] = \{1, 2, \dots, N\}$. **Configuration space** $\mathcal{S}_N = \{-1, +1\}^N$
Configuration $\sigma = (\sigma_i)_{i \in [N]} \in \mathcal{S}_N$, $\sigma_i \in \{-1, +1\}$
$h > 0$ constant magnetic field.
Hamiltonian in the **randomly dilute Curie–Weiss model** (RDCW)

$$H_N(\sigma) = -\frac{1}{Np} \sum_{1 \le i < j \le N} J_{ij} \sigma_i \sigma_j - h \sum_{i \in [N]} \sigma_i$$

where $\{J_{ij}\}_{i,j \in [N]}$ is a sequence of i.i.d. random variables such that $J_{ij} = J_{ji}$ and $\mathbb{E}(J_{ij}) = p \in (0,1)$ constant [e.g. $J_{ij} \sim$ Ber(p)]

Relationship with the standard **Curie–Weiss model** (CW)

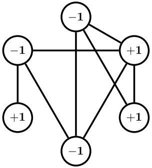
$$H_N^{\mathsf{CW}}(\sigma) = -\frac{1}{N} \sum_{1 \le i < j \le N} \sigma_i \sigma_j - h \sum_{i \in [N]} \sigma_i = \mathbb{E}(H_N(\sigma))$$

Graphical representation of configurations

Define the **interaction graph** $G = ([N], E) : (i,j) \notin E \iff J_{ij} = 0$

$$H_N(\sigma) = -\frac{1}{Np} \sum_{1 \le i < j \le N} J_{ij} \sigma_i \sigma_j - h \sum_{i \in [N]} \sigma_i$$

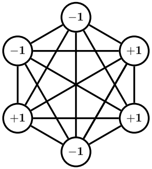
$$= -\frac{1}{Np} \sum_{\{i,j\} \in E} \sigma_i \sigma_j - h \sum_{i \in [N]} \sigma_i$$



We take $J_{ij} \sim \text{Ber}(p)$, $p \in (0,1) \implies G$ is an **Erdős–Rényi random graph** with fixed edge probability p and our model Curie–Weiss on Erdős–Rényi random graph

Standard Curie–Weiss model $\implies G$ is a complete graph

$$H_N^{\text{CW}}(\sigma) = -\frac{1}{N} \sum_{1 \le i < j \le N} \sigma_i \sigma_j - h \sum_{i \in [N]} \sigma_i$$



Consider again discrete time **Glauber dynamics** on $\mathcal{S}_N$ with equilibrium measure as reversible measure.

Background

Equilibrium RDCW model:

- Bovier and Gayrard, '93: prove that the RDCW free energy converges to that of the CW model (in the thermodynamic limit), when p decreases with the system size in a certain way.
- De Sanctis and Guerra, '08-'09: give an exact expression of the free energy first in the high temperature/low connectivity regime, and then at zero temperature (both annealed and quenched). Plus control of the fluctuations of the magnetisation in the high temperature limit.
- Kabluchko, Löwe and Schubert, '19: prove a quenched Central Limit Theorem for the magnetisation in the high temperature regime.
- Dembo, Montanari, '10; Dommers, Giardiná, van der Hofstad, '10; ...: extensive study of the Ising model on different kinds of random graphs.

Background

Metastability for interacting particle systems on random graphs:

- Dommers, den Hollander, Jovanovski, and Nardi, '17: random regular graph and configuration model with Glauber dynamics, in the limit as $\beta \to \infty$ and the number of vertices is fixed
- den Hollander and Jovanovski, '20: Erdős–Rényi random graph for fixed temperature in the $N \to \infty$ limit. It is exactly the RDCW model.

Results: metastable exit time for the RDCW model

Last exit-biased distribution

$$\nu_{A,B}(\sigma) = \frac{\mu_N(\sigma)\mathbb{P}_\sigma(\tau_B < \tau_A)}{\sum_{\sigma\in A}\mu_N(\sigma)\mathbb{P}_\sigma(\tau_B < \tau_A)}, \qquad \sigma \in A$$

Notation: $\nu_{m_-,m_+} = \nu_{\mathcal{S}_N[m_-(N)],\mathcal{S}_N[m_+(N)]}$

$\mathbb{P}_J$ is the law of the random couplings (or the law of the ER random graph).

Theorem (A.Bovier, S.Marello, E.P.)

For $\beta > 1$, $h > 0$ small enough and for $s > 0$, there exist absolute constants $k_1, k_2 > 0$ and $C_1(p,\beta) < C_2(p,\beta,h)$ independent of N, such that

$$\lim_{N\uparrow\infty} \mathbb{P}_J\left(C_1 \mathrm{e}^{-s} \le \frac{\mathbb{E}_{\nu_{m_-,m_+}}\left[\tau_{\mathcal{S}_N[m_+(N)]}\right]}{\mathbb{E}^{CW}_{m_-(N)}\left[\tau_{m_+(N)}\right]} \le C_2\mathrm{e}^{s}\right) \ge 1 - k_1\mathrm{e}^{-k_2 s^2}.$$

[*A. Bovier, S. Marello, and E. P., "Metastability for the dilute Curie–Weiss model with Glauber dynamics", Electron. J. Probab. 26: 1-38, 2021*]

Results: discussion

Theorem (A.Bovier, S.Marello, E.P.)

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$$\alpha = \left(\frac{\beta}{2p}\right)^2 \mathsf{Var}(J_{i,j}) = \frac{\beta^2(1-p)}{4p},$$

$$\kappa = \alpha + \epsilon(\alpha), \qquad \epsilon(\alpha) = \max_{\eta\in(0,1)} \left\{ \log\eta - \frac{\beta\sqrt{2\alpha + \log\left(\frac{c_1}{(1-\eta)^2}\right)}}{p\sqrt{2c_2}} \right\} < 0,$$

where $c_1, c_2 > 0$ are absolute constants and

$$C_1 = \mathrm{e}^{-2\beta-\alpha+\kappa}(1+o(1)), \qquad C_2 = \mathrm{e}^{2\beta(1+h)+2\alpha}(1+o(1)).$$

Results: discussion

Theorem (A.Bovier, S.Marello, E.P.)

For $\beta > 1$, $h > 0$ small enough and for $s > 0$, there exist absolute constants $k_1, k_2 > 0$ and $C_1(p,\beta) < C_2(p,\beta,h)$ independent of N, such that

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Comparison with den Hollander and Jovanovski:
With $\mathbb{P}_J \to 1$ as $N \to \infty$,

$$\mathbb{E}_\xi\left[\tau_{\mathcal{S}_N[m_+(N)]}\right] = N^{\mathcal{E}_N} \exp\Big(\beta N \left[F_\beta(m^*) - F_\beta(m_-)\right]\Big),$$

uniformly in $\xi \in \mathcal{S}_N[m_-(N)]$ (!). They prove that the multiplicative error term is at most *polynomial* in N, but do not know how to identify the *random* prefactor. They use pathwise approach to metastability and obtain also the exp law result.

Potential theoretic approach (Bovier, Eckhoff, Gayrard and Klein, 2001)

Translates the problem of understanding the metastable behaviour of Markov processes to the study of capacities of electric networks. Link between **mean metastable crossover time** and **capacity**.

For A, B disjoint subsets of $\mathcal{S}_N$, the key formula is

$$\mathbb{E}_{\nu_{A,B}}[\tau_B] = \sum_{\sigma \in A} \nu_{A,B}(\sigma)\mathbb{E}_\sigma[\tau_B] = \frac{1}{\mathrm{cap}(A,B)} \sum_{\sigma' \in \mathcal{S}_N} \mu_N(\sigma')h_{A,B}(\sigma'),$$

where

$$\mathrm{cap}(A,B) = \sum_{\sigma \in A} \mu_N(\sigma)\mathbb{P}_\sigma(\tau_B < \tau_A)$$

and $h_{A,B}$ is called *harmonic function*

$$h_{A,B}(\sigma) = \begin{cases} \mathbb{P}_\sigma(\tau_A < \tau_B) & \sigma \in \mathcal{S}_N \setminus (A \cup B), \\ \mathbb{1}_A(\sigma) & \sigma \in A \cup B. \end{cases}$$

Idea

Estimate capacity and harmonic function in terms of those of the CW model

Concentration inequalities and closeness to CW

The key point in order to find sharp bounds for the capacity is the following result (ideas from Talagrand, proof in the talk of Saeda)

Concentration

There exist $k_1, k_2 \geq 0$ such that, for any $g : \mathcal{S}_N \to [0, \infty)$ and any $s > 0$

$$\mathbb{P}_J \left(\mathrm{e}^{-s+\kappa} \leq \frac{\sum_{\sigma \in \mathcal{S}_N} g(\sigma) \mathrm{e}^{-\beta(H_N(\sigma) - \mathbb{E}[H_N(\sigma)])}}{\sum_{\sigma \in \mathcal{S}_N} g(\sigma)} \leq \mathrm{e}^{s+\alpha} \right) \geq 1 - k_1 \mathrm{e}^{-k_2 s^2}$$

asymptotically for $N \to \infty$.

Concentration inequalities and closeness to CW

Concentration

There exist $k_1, k_2 \geq 0$ such that, for any $g : \mathcal{S}_N \to [0, \infty)$ and any $s > 0$

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asymptotically for $N \to \infty$.

Mesoscopic measure:

$$Z_N \mathcal{Q}_N(m) = Z_N \sum_{\sigma \in \mathcal{S}_N[m]} \mu_N(\sigma) = \sum_{\sigma \in \mathcal{S}_N[m]} \mathrm{e}^{-\beta(H_N(\sigma) - \mathbb{E}[H_N(\sigma)])} \mathrm{e}^{-\beta \mathbb{E}[H_N(\sigma)]}$$

Choose $g(\sigma) = \mathrm{e}^{-\beta \mathbb{E}[H_N(\sigma)]} \mathbf{1}_{\sigma \in \mathcal{S}_N[m]}$, and recall that $\mathbb{E}[H_N(\sigma)] = NE(m)$ and $Z_N^{\mathsf{CW}} \mathcal{Q}^{\mathsf{CW}}{}_N(m) = \mathrm{e}^{-\beta\, NE(m)} |\mathcal{S}_N[m]|$

$$\lim_{N \to \infty} \mathbb{P}_J \left(\mathrm{e}^{-s+\kappa} \leq \frac{Z_N \mathcal{Q}_N(m)}{Z_N^{\mathsf{CW}} \mathcal{Q}_N^{\mathsf{CW}}(m)} \leq \mathrm{e}^{s+\alpha} \right) \geq 1 - k_1 \mathrm{e}^{-k_2 s^2}$$

Capacity estimates

We are interested in $\mathbb{E}_{\nu_{A,B}}[\tau_B] = \dfrac{1}{\boxed{\mathrm{cap}(A,B)}} \sum_{\sigma' \in \mathcal{S}_N} \mu_N(\sigma') h_{A,B}(\sigma')$

with $A = \mathcal{S}_N[m_-(N)], B = \mathcal{S}_N[m_+(N)]$

Theorem (Bounds for the capacity)

For any $m_1 \neq m_2 \in \Gamma_N$ and any $s > 0$, there exist absolute constants $k_1, k_2 > 0$ such that asymptotically as $N \to \infty$

$$\mathbb{P}_J \left(c_1 \mathrm{e}^{-(s+\alpha)} \leq \frac{Z_N \,\mathrm{cap}\,(\mathcal{S}_N[m_1], \mathcal{S}_N[m_2])}{Z_N^{CW} \,\mathrm{cap}^{CW}(\mathcal{S}_N[m_1], \mathcal{S}_N[m_2])} \leq c_2 \mathrm{e}^{s+\alpha} \right) \geq 1 - k_1 \mathrm{e}^{-k_2 s^2}$$

where c_1, c_2 are explicit and depend on β, h.

Proof: Upper bound via Dirichlet principle, lower bound via Thomson principle.

Variational principles for capacity estimates

Dirichlet principle

$$\mathrm{cap}(A,B) = \inf_{f\in\mathcal{H}_{AB}} \mathcal{E}(f) = \inf_{f\in\mathcal{H}_{AB}} \frac{1}{2} \sum_{\sigma,\sigma'\in\mathcal{S}_N} \mu_N(\sigma) p_N(\sigma,\sigma')[f(\sigma)-f(\sigma')]^2.$$

$$\mathcal{H}_{AB} := \{f : \mathcal{S}_N \to [0,1] : f|_A = 1, f|_B = 0, \mathcal{E}(f) < \infty\}$$

Variational principles for capacity estimates

Let E be the set of pairs (x, y) such that $p_N(x, y) \neq 0$, $x, y \in \mathcal{S}_N$.
Then a unit flow is a map $\varphi : E \to \mathbb{R}$ such that

$$\sum_{\substack{y \in V: \\ (y,x) \in E}} \varphi(y, x) = \sum_{\substack{w \in V: \\ (x,w) \in E}} \varphi(x, w) \qquad \text{(Kirchhoff's law)}$$

$$\sum_{a \in A} \sum_{\substack{y \in V: \\ (a,y) \in E}} \varphi(a, y) = 1 = \sum_{b \in B} \sum_{\substack{y \in V: \\ (y,b) \in E}} \varphi(y, b).$$

$\mathcal{U}_{AB}$ is the space of all unit flows from A to B

Thomson principle

$$\mathrm{cap}(A, B) = \sup_{\phi \in \mathcal{U}_{AB}} \frac{1}{\mathcal{D}(\phi)},$$

where

$$\mathcal{D}(\phi) = \sum_{(\sigma,\sigma') \in E} \frac{\phi(\sigma, \sigma')^2}{\mu_N(\sigma) p_N(\sigma, \sigma')}$$

Concentration inequalities and capacity estimates

Estimates on the harmonic function

We are interested in $$\mathbb{E}_{\nu_{A,B}}[\tau_B] = \frac{1}{\mathrm{cap}(A,B)} \boxed{\sum_{\sigma' \in \mathcal{S}_N} \mu_N(\sigma') h_{A,B}(\sigma')}$$

with $A = \mathcal{S}_N[m_-(N)], B = \mathcal{S}_N[m_+(N)]$. Recall

$$h_{A,B}(\sigma) = \begin{cases} \mathbb{P}_\sigma(\tau_A < \tau_B) & \sigma \in \mathcal{S}_N \setminus (A \cup B), \\ \mathbb{1}_A(\sigma) & \sigma \in A \cup B. \end{cases}$$

Notation $h^N_{m_-,m_+} = h_{A,B}$.

Theorem (Upper bound for the harmonic sum)

For any $s > 0$, there exist absolute constants $k_1, k_2 > 0$ such that

$$\mathbb{P}_J\left(\sum_{\sigma \in \mathcal{S}_N} \mu_N(\sigma) h^N_{m_-,m_+}(\sigma) \le \mathrm{e}^{\alpha+s} \frac{\exp\left(-\beta N F(m_-)\right)}{Z_N \sqrt{(1-m_-^2)\beta F''(m_-)}} (1+o(1))\right)$$
$$\ge 1 - k_1 \mathrm{e}^{k_2 s^2}$$

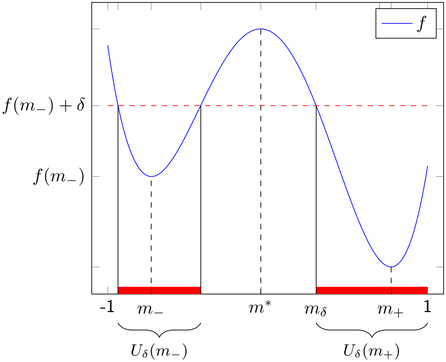
The lower bound is similar, replacing $\mathrm{e}^{\alpha+s}$ with $\mathrm{e}^{\kappa-s}$.

Estimates on the harmonic function

To estimate

$$\sum_{\sigma \in \mathcal{S}_N} \mu_N(\sigma) h^N_{m_-,m_+}(\sigma)$$

decompose the configuration space $\mathcal{S}_N$



$$\boxed{\mathcal{S}_N = \mathcal{S}_N\left[U_\delta(m_-)\right] \cup \mathcal{S}_N\left[U_\delta^c\right] \cup \mathcal{S}_N\left[U_\delta(m_+)\right]}$$

The main contribution is given by $\mathcal{S}_N[U_\delta(m_-)]$. Indeed, μ_N is very small in $\mathcal{S}_N[U_\delta^c]$ while $h^N_{m_-,m_+}$ is very small in $\mathcal{S}_N[U_\delta(m_+)]$. We follow [1]

[1]A. Bianchi, A. Bovier, and D. Ioffe. Sharp asymptotics for metastability in the random field Curie-Weiss model. *Electron. J. Probab.*, 2009.

Related problems

- Metastability for the dilute Curie-Weiss Potts model: spins are in $\{1, ..., q\}$ (together with Johan Dubbeldam, Vicente Lenz and Martin Slowik);
- Metastability for the dilute Hopfield model: couplings $\epsilon_{i,j} J_{i,j}$ where $J_{i,j} = \langle \xi_i, \xi_j \rangle$ are "patterns" and $\epsilon_{i,j}$ are i.i.d.RVs (together with Saeda Marello and Martin Slowik);
- Metastability on inhomogeneous random graphs (ongoing Anton Bovier, Frank den Hollander and Saeda Marello), talk Saeda.

Thank you for your attention!