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Morphological approach to extract ridge and valley connectivity networks from Digital Elevation Models

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Abstract. The extraction of ridge and valley connectivity networks is essential for studying spatio-temporal organizations. Extraction of such connectivity networks from multiscale DEMs has lately received notable attention. A simple method is proposed to extract these networks, from a sample DEM as well as a simulated fractal DEM, using non-linear morphological transformations in a methodical way. Further, the proposed method can be adapted to extract these complex topological networks from DEMs generated from either remotely sensed or topographic data.

1. Introduction

The two major network types with physical expression in a landscape are channel and ridge networks (Mark 1988). Ridges also form networks as channels. These two networks often have very similar lengths. Ridges in a fluvially eroded landscape are those parts of the original landmass that have been eroded the least. The ridges can be defined in several ways (Mark 1981, 1988). The ridge acts as a barrier that obstructs further diffusion of water flow from valley. The theoretical studies on ridge topology are initiated by several scientists (Cayley 1859, Maxwell 1870, Gilbert 1877, Davis 1899, Mark 1979). Spatial pattern of ridges and valleys (figure 1), and processes are clearly (perpetually) intertwined due to the fact that the geomorphic processes are primarily dictated by exogenic and endogenic nature of processes that further influence the spatial organization. To understand spatial organization of the ridge—valley connectivity networks, one must be able to classify patterns according to their spatial distribution, and to develop process based models that can produce observed patterns at multiscales. An attempt was made in this regard (Sagar *et al.* 2000), wherein only binary morphological transformations and certain logical operations have been used. The present scheme aims to develop mathematical framework based

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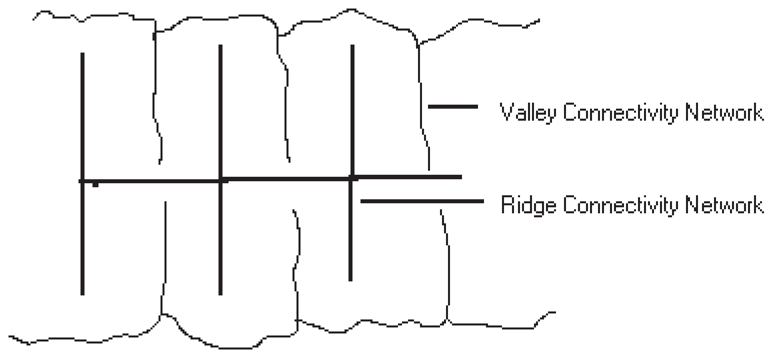


Figure 1. Spatial patterns of valley and ridge connectivity networks.

on grey level morphological transformations to extract the ridge and valley connectivity networks automatically from DEMs. The extracted connectivity networks help to understand spatial distribution and to classify the region according to non-uniformity in the spatial distribution.

The proposed method to extract the ridge and valley connectivity networks from multiscale DEM data is given in section 2. In section 3, two sample cases are considered to test the proposed method. The use of the results for developing cogent geomorphic models is discussed.

2. Algorithm to extract ridge-valley connectivity networks

A mathematical framework to automatically extract ridge and valley connectivity networks from DEMs is presented in this section. The basic notations, definitions of morphological transformations, and sequential steps of the algorithm are described.

2.1. *Basic grey level morphological transformations*

Dilation, erosion, opening and closing (Serra 1982) are the simplest quantitative morphological set transformations. The discrete grey level image, M, is defined as a finite subset of Euclidean two dimensional space, lR^2 that can have values between 0 and 255. The *supremums* and *infimums* in the regions of elevations represented in grey levels, of DEM, hereafter referred to as M, testify the presence of zones of ridges and valleys. To identify ridge and valley connectivity networks, the following method is adapted. Morphological dilations and erosions are transformations that expand and contract a set. These transformations can be visualized as working with two images: the image being processed (M), and a structuring template (S). Each structuring template has a designed shape that acts as a probe. The image (M) can be decomposed by probing it with various structuring templates to unravel certain complex features of topological nature. The four basic morphological transformations are defined with respect to the structuring element S, scaling factor e, image M and point $A_0 \in lR^2$.

$$\text{Dilation:} \quad \delta_s^e(M)(A_0) = MAX_{A \in A_0 + eS(A_0)}(M(A)) \tag{1}$$

Dilation enlarges the bright spots that represent the higher elevation regions in the

DEM and neighbouring grey levels of elevation regions will be connected.

$$\text{Erosion:} \quad \varepsilon_s^e(M)(A_0) = MIN_{A \in A_0 + eS(A_0)}(M(A)) \tag{2}$$

Erosion enlarges the dark spots that represent the zones of lower elevations or valleys.

The dilation followed by erosion is closing transformation, while erosion followed by dilation is opening transformation. These transformations can be carried out according to the multiscale approach (Serra 1982). In the multiscale approach, the size of the structuring template will be increased from iteration to iteration. The opening and closing are defined, respectively, as:

$$\text{Opening:} \quad \gamma_s^e(M) = \delta_s^e(\varepsilon_s^e(M)) \tag{3}$$

$$\text{Closing:} \quad \varphi_s^e(M) = \varepsilon_s^e(\delta_s^e(M)) \tag{4}$$

Due to the noninvertible properties of the two superficially simple cascade transformations, ridge and valley connectivity networks can be extracted.

2.2. *Algorithm*

Streams flow between the ridges. From physicist's words these ridges are Brownian motions that act as barriers (Takayasu 1990). However, in geomorphologist's terms, an elevation contour possesses crenulations of 'V' and 'Λ' shapes which can be inferred as the paths of ridge and valley connectivity networks. In mathematical terms these are called *supremums* and *infimums* that can be seen in DEM data. The morphological transformations described in previous section are systematically used to remove these *supremums* and *infimums*. These two topographically significant features enable the structural composition of the terrain. This structural composition is sensitive to any changes that occur due to both exogenic and endogenic processes. Whether underlain geological structures influence these surfacial features of geomorphic nature purely depends on the topologic composition of ridge and valley connectivity networks. Hence, it is appropriate to use the topology and set theory based techniques to exploit these disparities to derive meaningful conclusions in terms of extraction of complex topographic information. A framework based on nonlinear grey level morphological transformations to extract valley and ridge connectivity network information from the DEM is explained.

2.2.1. RCN *subsets of nth order*

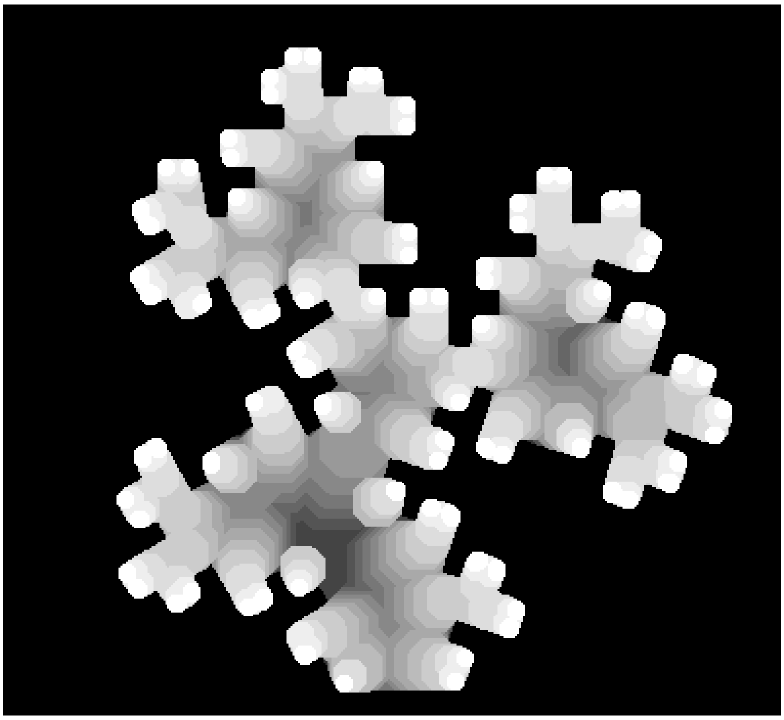
In the process of extraction of *RCN*s from the image, the image (M) will be considered. This process is mathematically shown as:

$$RCN_e(M) = [\varepsilon_s^e(M)] \backslash \{\gamma_s[\varepsilon_s^e(M)]\} \tag{5}$$

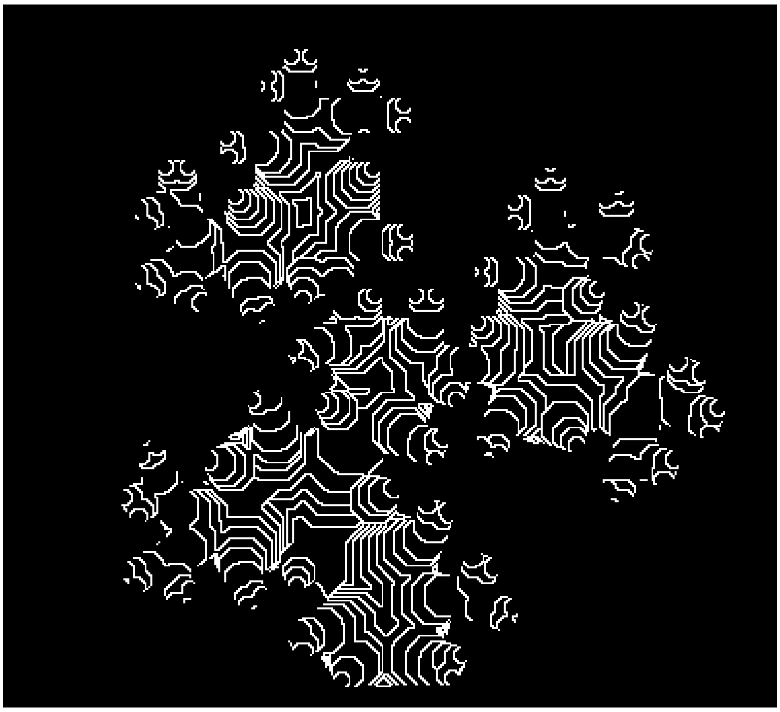
where $e = 1, 2, \ldots, n$.

2.2.2. VCN *subsets of nth order*

The valley connectivity network (VCN) is nothing but the network path of regional minima and can be isolated by applying the following transformations. The opened version of each eroded inverted-image ($M_{inverse}$) is subtracted from the corresponding eroded inverted-image. In the inverted image it is obvious that the high

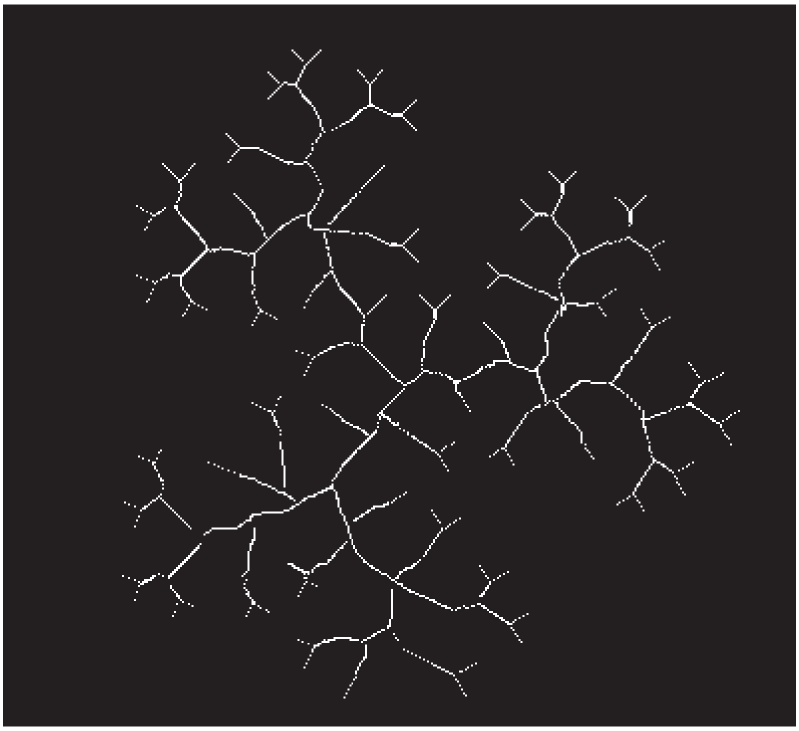


(*a*)

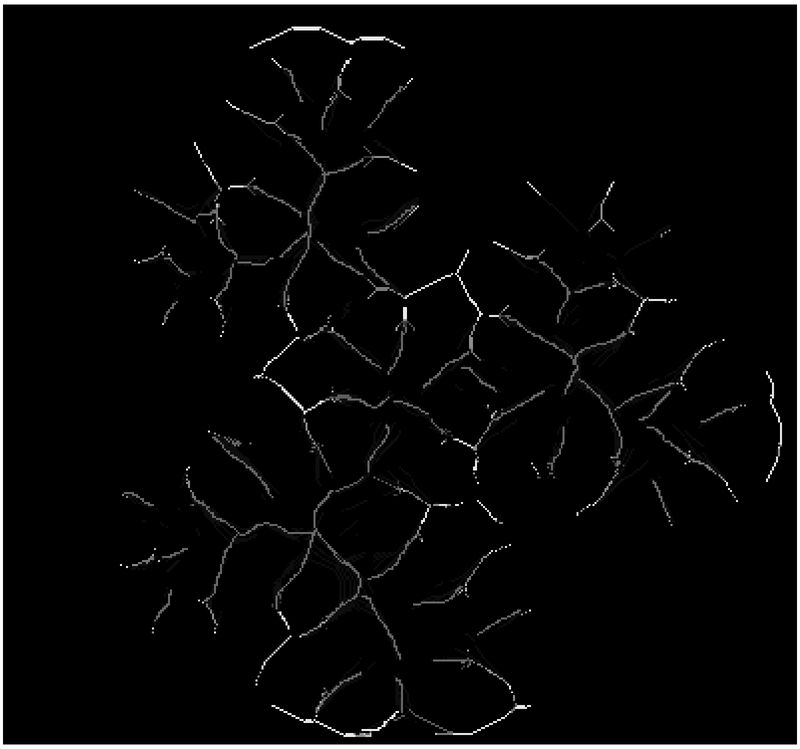


(*b*)

Figure 2. (*a*) A simulated fractal DEM; (*b*) elevation contours; (*c*) VCN; and (*d*) RCN.



(*c*)



(*d*)

and lower elevations will be represented respectively by darker and brighter grey scales. This process is mathematically shown as:

$$VCN_e(M) = [\varepsilon_s^e(M_{inverse})] \backslash \{\gamma_s[\varepsilon_s^e(M_{inverse})\} \quad (6)$$

where $e = 1, 2, \ldots, n$.

Once several order subsets of ridge and valley networks are isolated from respective scaled-images, union of subsets is made following the logical AND operation.

$$RCN(M) = \cup_{e=1}^{n}[RCN_e(M)] \quad (7)$$

$$VCN(M) = \cup_{e=1}^{n}[VCN_e(M)] \quad (8)$$

The framework thus developed extracts *RCN*s and *VCN*s from the multiscale DEM data.

3. Case study

In order to test the algorithm, two cases have been considered. The first one is a simulated DEM. A triangular initiator-basin is transformed as a fractal (Sagar *et al.* 2001) by following the principle involved in Koch curve generation. This binary fractal has been decomposed into topologically prominent regions (*TPR*s). These *TPR*s have been assigned coding assuming that the *TPR*s of specific grey level represents a spatially distributed region of a specific elevation. A detailed procedure to simulate a fractal basin may be seen in Sagar and Murthy (2000). The simulated fractal DEM, the elevation contours, and the valley and ridge connectivity networks thus extracted by following the proposed algorithm are shown, respectively, in figures 2a–d. The second case is used here; this is small area of Yellowstone DEM (figure 2a), the details of which can be seen at http://edcwww.cr.usgs.gov/glis/hyper/guide/usgs_dem.

The ridge and valley connectivity networks have been extracted from the multiscale DEM data, generated by applying the Gaussian blurring technique. As the blurring level increases the merge of small regions into surrounding higher grey level region is quite visible in figures 3(*a*), 4(*a*), 5(*a*), 6(*a*), and 7(*a*). As the spatial resolution increases, so does the intricacy of the connectivity networks, and vice versa. By applying the mathematical framework developed on the DEM data of various scales, valley and ridge network maps have been extracted, and are shown respectively in figures 3(*b*), 4(*b*), 5(*b*), 6(*b*), and 7(*b*), and 3(*c*), 4(*c*), 5(*c*), 6(*c*), and 7(*c*). The present method yields results faster than the method considered by Mark (1979). It is superficially simple but elegant framework that is based on fundamental grey level morphological transformations. It is observed that the computed mono-fractal dimensions (table 1) of *RCN*s and *VCN*s of all spatial scales are directly proportional to each other. Such results may envisage new insights as a first step to understand the complex geomorphological processes.

The real world DEMs, which are arrays of numbers that represent spatial distribution of terrain altitudes, can be derived indirectly from digitized topographic maps or directly through photogrammetric processing of aerial photographs and information gathered remotely by sensors with stereo viewing capability (Franklin 1990). These DEMs contain crenulations of various degrees in the transition between the two successive erosion frontlines of each spatially distributed elevation region. The straightforward method developed in this study benefits from the advantage of these crenulations, which are present in each successive erosion frontline of all the spatially distributed elevation regions. Therefore, this method can be adapted to any raster

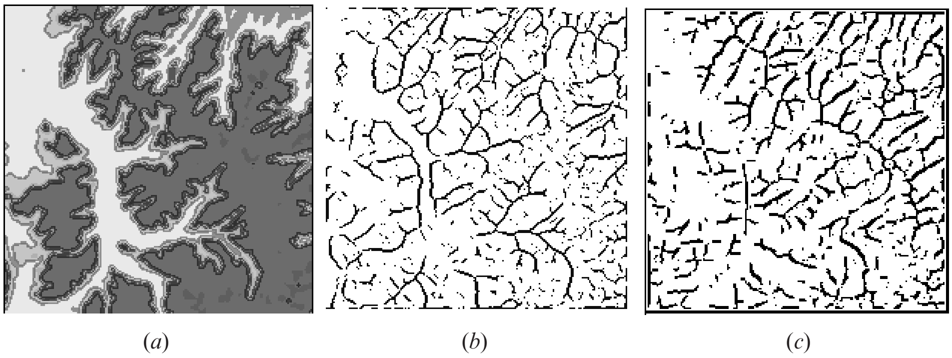


Figure 3. (*a*) A sample DEM of size 200 × 200 pixels of a small part of US (downloaded from the Internet) after Gaussian blurring with 1-pixel radius; (*b*) valley connectivity network (VCN); and (*c*) ridge connectivity networks (RCN).

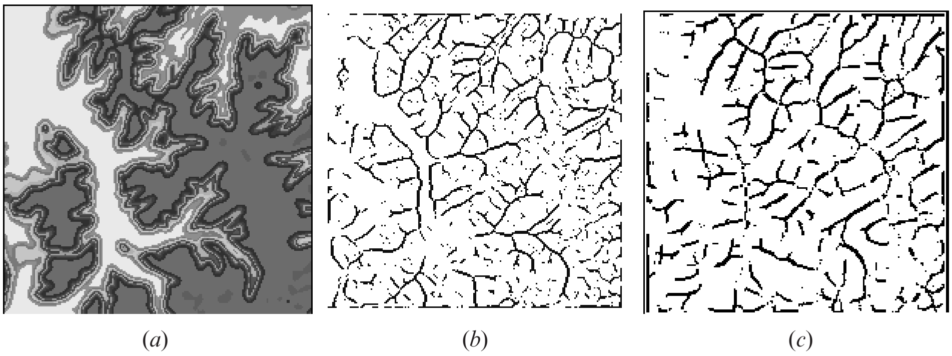


Figure 4. (*a*) DEM after Gaussian blurring with a 2-pixel radius; (*b*) VCN; and (*c*) RCN.

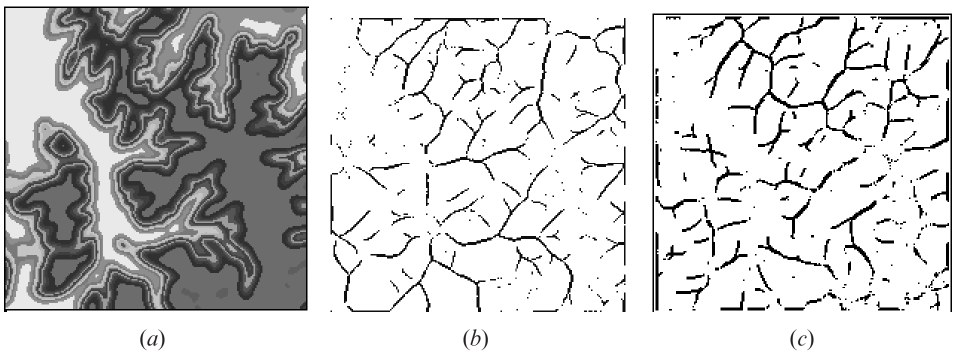
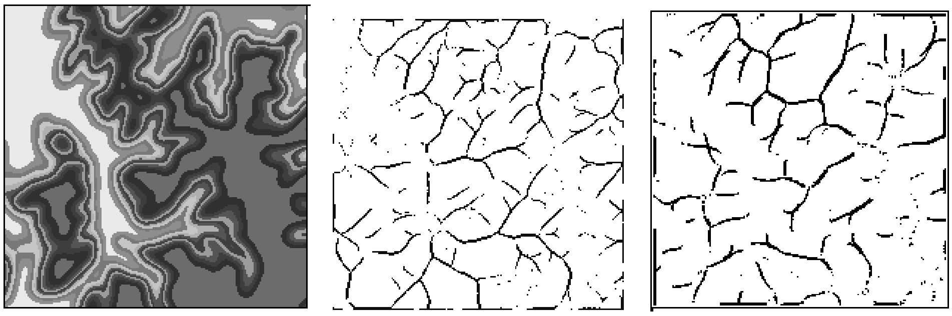


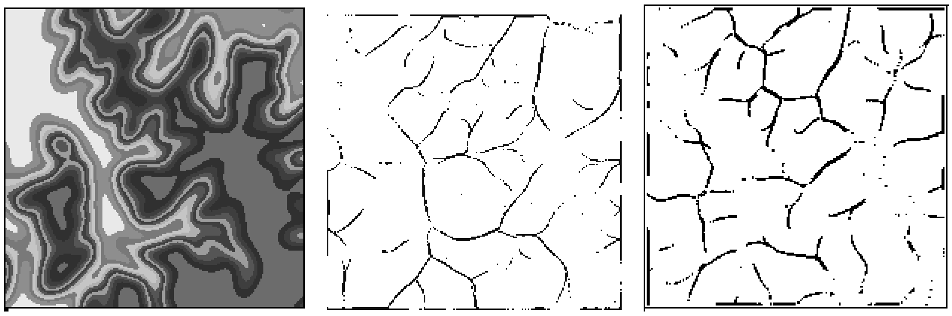
Figure 5. (*a*) DEM after Gaussian blurring with a 3-pixel radius; (*b*) VCN; and (*c*) RCN.

based DEM that can be derived by either direct or indirect methods. However, due to unavailability of DEM of a rugged terrain that generally consists of sufficiently large number of crenulations of several degrees to expect an intricate channel and ridge connectivity networks, a transcendentally generated DEM, and a small piece of Yellowstone DEM have been considered to test this morphology based framework.



(*a*) (*b*) (*c*)

Figure 6. (*a*) DEM after Gaussian blurring with a 4-pixel radius; (*b*) VCN; and (*c*) RCN.



(*a*) (*b*) (*c*)

Figure 7. (*a*) DEM after Gaussian blurring with a 5-pixel radius; (*b*) VCN; and (*c*) RCN.

In contrast, this method takes more number of iterations to extract all possible subsets of ridge and channel networks from a DEM of lesser rugged terrain.

Once the error free multiscale DEMs in a time sequential mode are available, the extracted abstract topographic information can be used to understand the spatio-temporal organization. As the subtle morphological changes can be recorded in the valley and ridge connectivity networks that are apparent in the interferometrically generated multi-temporal DEMs, one can better quantify the spatio-temporal organization of these features of geomorphic interest by using concepts from multifractal formalism. The proposed method can be adapted with DEM generated from data obtained through remote sensing methods also. Multifractal measures (Mandelbrot

Table 1. Box-counting dimension of multiscale valley connectivity networks (VCNs) and ridge connectivity networks (RCNs) extracted from a sample DEM.

Gaussian blur level (in pixel(s) radius)	VCNs	RCNs
1	1.7895	1.7819
2	1.7295	1.7320
3	1.6364	1.6520
4	1.5792	1.5653
5	1.5157	1.5152

1982, Halsey *et al.* 1986) may be computed for the ridge and valley connectivity networks thus extracted from multiscale DEMs, so that further comparisons can be made between the estimated multifractal measures for the sequence of ridge, and valley connectivity network maps extracted from the multi-temporal DEMs, in order to quantify the degree of heterogeneity in the spatio-temporal organization. Our future work plan is to compute these multifractal measures for the *RCN*s and *VCN*s extracted from multi-scale temporal DEMs of varied geological terrains, in order to understand the spatio-temporal organization in a firm quantitative manner.

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