June 12th, 2019 **SWMS-2018-Follow up**

Worksheet 5 **matrices review**

1. Find solutions of the following system of linear equations in four variables using row-reduction.

$$\begin{aligned} x+2y-3z+w &= -2 \\ 3x-y-2z-4w &= 1 \\ 2x+3y-5z+w &= -3 \end{aligned}$$

2. Consider the vectors

$$u = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \quad \text{and} \quad w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Determine whether these vectors are linearly independent or linearly dependent.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{bmatrix}.$$

(a) Find the column space of A. Compute a basis for the column space of A.
(b) Find the null space of A. Compute a basis for the null space of A.

4. Calculate the dimension of the column space (or *rank*) of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 4 & 2 & 3 \end{bmatrix}$$

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5. Consider the three planes

$$x+2y+5z=7 \qquad 2x-y=-1 \qquad 2x+y+4z=k$$

(a) For which values of the parameter k do these three planes have at least one point in common?
(b) Determine the common points.

6. Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.
(a) Find the eigenvalues of A. Find the corresponding eigenvectors.
(b) Find a basis for the corresponding eigenspaces.

7. Show that te following statements are equivalent:
(a) There exists an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.
(b) A has a basis consisting of eigenvectors.

8. Show that 0 is an eigenvalue of an $n \times n$ matrix A if and only if A is a singular matrix, ie., $rank(A) < n$.

9. Let A be an $n \times n$ real orthogonal matrix. Then show that
(a) The rows of A will form an orthonormal basis of $\mathbb{R}^n$.
(b) The columns of A will form an orthonormal basis of $\mathbb{R}^n$.

10. Find a basis for the space of symmetric $n \times n$ matrices.