June 11th, 2019 **SWMS-2018-Follow up**

Worksheet 4 **symmetric groups, class equation**

1. Write down the cycle decomposition of all the elements of S_4. Find their orders.
2. Show that S_n is non-abelian for all $n \geq 3$.
3. Find the conjugacy classes of Q_8.
4. Show that disjoint cycles commute.
5. Show that if σ, τ are elements of the symmetric group S_n and suppose σ has cycle decomposition
$$(a_1 a_2 \dots a_{k_1})(b_1 b_2 \dots b_{k_2}) \dots$$
then $\tau\sigma\tau^{-1}$ has cycle decomposition
$$(\tau(a_1)\tau(a_2) \dots \tau(a_{k_1}))(\tau(b_1)\tau(b_2) \dots \tau(b_{k_2})) \dots.$$
6. Suppose that the cycles σ_1, σ_2 in S_n have the same cycle type. Show that they are conjugate elemets in S_n.
7. Show that there is a bijection between conjugacy classes of S_n and partitions of n.
8. Let $\sigma_1 = (1)(35)(89)(2476)$ and let $\sigma_2 = (4)(12)(36)(5789)$. Find a τ such that $\tau\sigma_1\tau^{-1} = \sigma_2$.