1. For $\{x_n\}_{n=1}^{\infty}$given in each of the following, please compute $\frac{x_{n+1}}{x_n}$
   (a) $x_n = \frac{1}{n}$
   (b) $x_n = \frac{2^n}{n!}$,
   (c) $x_n = nb^n$, for $b \in (0, 1)$
   (d) $x_n = \frac{n!}{n^n}$
   (e) $x_n = \frac{n!}{n^n}$
   (f) $x_n = \frac{(n!)^2}{n^n}$
   (g) $x_n = \frac{2n!}{n^{2n}}$
   (h) $x \in \mathbb{R}$ and for $n \geq 0$, $x_n = \frac{x^n}{n!}$

   By providing a bound on $\frac{x_{n+1}}{x_n}$, can you determine if $\sum_{n=1}^{\infty} x_n$ is a positive real number.

2. For $\{y_n\}_{n=1}^{\infty}$given in each of the following, please compute $y_n^{\frac{1}{n}}$
   (a) $y_n = \left(\frac{n}{2n+1}\right)^n$
   (b) $y_n = \frac{2^n}{n!}$
   (c) $x \in \mathbb{R}$ and for $n \geq 0$, $y_n = n^n x^n$
   (d) $y_n = \begin{cases} \frac{1}{2^k} & \text{if } n = 2k-1, k \geq 1 \\ \frac{1}{3^k} & \text{if } n = 2k, k \geq 1 \end{cases}$
   (e) $y_n = \begin{cases} \frac{1}{2^{2k-1}} & \text{if } n = 2k-1, k \geq 1 \\ \frac{4}{2^{2k}} & \text{if } n = 2k, k \geq 1 \end{cases}$

   By providing a bound on $y_n^{\frac{1}{n}}$, can you determine if $\sum_{n=1}^{\infty} y_n$ is a positive real number.

3. For $\{z_n\}_{n=1}^{\infty}$given in each of the following, decide if you can identify $\lim_{n\to\infty} z_n$.
   (a) $z_n = \frac{1}{n}$
   (b) $z_n = \frac{3}{5^{\frac{1}{n}}+6}$
   (c) $z_n = \frac{(-1)^{n-1}n}{3n-2}$

   Based on the answer, can you determine if $\sum_{n=1}^{\infty} z_n$ is a real number.

4. Let $x \in \mathbb{R}$. Decide the particular range of x where the series $\sum_{n=0}^{\infty} z_n$ converges in each of the following cases:
   (a) $z_n = \frac{n!}{n^n} x^n$
   (b) $z_n = \frac{(n!)^2}{n^n} x^n$
   (c) $z_n = \frac{n!}{n^n} (x-2)^n$
   (d) $z_n = \frac{(4x-12)^n}{(-3)^{2+n}(n^2+1)}$

1. Let $y_1, y_2 \in \mathbb{R}$ be given, and define recursively for $n \geq 1$,

$$y_{n+2} = \frac{1}{3}y_n + \frac{2}{3}y_{n+1}$$

for all $n \in \mathbb{N}$. Show from definition that the sequence is a Cauchy sequence.

2. Consider the $\{y_n\}_{n=1}^{\infty}$, such that $y_1 > 1$ and $y_{n+1} := 2 - \frac{1}{y_n}$ for $n \geq 2$. Decide whether the sequence y_n converges.

3. Let $x \in \mathbb{R}$, $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers such that $x_n \to x$ as $n \to \infty$.
   (a) Let $\{y_n\}_{n=1}^{\infty}$be another sequence such that $y_n = x_n$ for all $n \geq K$. Does it imply that y_n also converges ?
   (b) $\forall \epsilon \in (0, 1)$, there is an $N \in \mathbb{N}$ such that $| x_n - x |< \epsilon$ for all $n \geq N$.
   (c) Let $C > 0$ and $\{y_n\}_{n=1}^{\infty}$be another sequence such that $y_n = Cx_n$ for all $n \geq 1$. Does it imply that y_n also converges ?
   (d) Let $\{y_n\}_{n=1}^{\infty}$be another sequence such that $y_n = (-1)^n x_n$ for all $n \geq 1$. Does it imply that y_n also converges ?

4. Let $p \in \mathbb{R}$. Decide whether $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{(n+1)^p}$ converges.

5. Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence. Then it has a subsequence convergent in $\mathbb{R}$. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers.