- **(Sample Space)** A sample space S is a set. The elements of the set S will be called "outcomes" and should be viewed as a listing of all possibilities that might occur. We will call the process of actually selecting one of these outcomes an "experiment".
- **(Temporary Definition of Event)** Given a sample space S, an "event" is any subset $E \subset S$.
- **(Probability Space Axioms)** Let S be a sample space and let $\mathcal{F}$ be the collection of all events. A "**probability**" is a function $P : \mathcal{F} \to [0, 1]$ such that:
  (1) $P(S) = 1$; and
  (2) If $E_1, E_2, ...$ are a countable collection of disjoint events (that is, $E_i \cap E_j = \varnothing$ if $i \neq j$), then $P(\bigcup_{j=1}^{\infty} E_j) = \sum_{j=1}^{\infty} P(E_j)$.

1. We toss a coin and note down the outcome. Write down the sample space S for this experiment. A coin is considered "fair" if each of these outcomes is equally likely. Which axioms or properties above can be used to reach the (obvious) conclusion that both outcomes have a 50% chance of occurring?
2. A jar contains a large collection of red, green, and white marbles. Marbles are drawn from the jar one at a time. The color of the marble is recorded and it is put back in the jar before the next draw. Let R_n denote the event that the n^{th} draw is a red marble and let G_n denote the event that the n^{th} draw is a green marble. For example, $R_1 \cap G_2$ would denote the event that the first marble was red and the second was green. In terms of these events (and appropriate set-theoretic symbols – union, intersection, and complement) find expressions for the events in parts (a), (b), and (c) below.
   (a) The first marble drawn is white. (We might call this W_1, but show that it can be written in terms of the R_n and G_n sets described above).
   (b) The first marble drawn is green and the second marble drawn is not white.
   (c) The first and second draws are different colors.
   (d) Let $E = R_1 \cup G_2$ and let $F = R_1^c \cap R_2$. Are E and F disjoint? Why or why not?
3. Let A and B denote events in a Sample Space S. Let P be a probability on S. Complete the following table:

$P(A)$	$P(B)$	$P(A \cup B)$	$P(A \cap B)$	$P(A - B)$	$P(B - A)$	$P(A \triangledown B)$
0.7	0.6	0.9				
			0.1	0.3	0.2	
	0.8	1	0			
0.6	0.4		0.4			
0.5	0.5	0.5				
	0.6		0.2	0.1		
		0.5	0	0.3		
		0.8		0.4	0.1	
0	0.3					
1	0.4					
0.8			0.5			0.4
0.4		0.7				0.5

4. Two dice are rolled. How likely is it that their sum will equal eight ?

Let S be a sample space with probability P. Let A and B be two events with $P(B) > 0$. Then the conditional probability of A given B written as $P(A|B)$ and is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

1. Suppose we have coloured balls distributed in three boxes in quantities as given by the table below:

	Box 1	Box 2	Box 3
Red	4	3	3
Green	3	3	4
Blue	5	2	3

   A box is selected at random. From that box a ball is selected at random. How likely is it that a red ball is drawn?

2. A probability class has fifteen students - four seniors, eight juniors, and three sophomores. Three different students are selected at random to present homework problems. What is the probability the selection will be a junior, a sophomore, and a junior again, in that order?

3. Shyam is randomly selected from the citizens of Hyderabad by the Health authorities. A laboratory test on his blood sample tells Shyam that he has tested positive for Swine Flu. It is found that 95% of people with Swine Flu test positive but 2% of people without the disease will also test positive. Suppose that 1% of the population has the disease. What is the probability that Shyam indeed has the Swine Flu ?

4. (**Polya's Urn Scheme**) Suppose there is an urn that contains r red balls and b black balls. A ball is drawn at random and its colour noted. It is replaced with $c > 0$ balls of the same colour. The procedure is then repeated. For $j = 1, 2, \ldots$, let R_j and B_j be the events that the j-th ball drawn is red and black respectively. What is $P(R_1)$ and $P(R_2)$?