June 6th, 2019 **SWMS-2018-Follow up**
Worksheet 1 **Series**

1. In each of the cases below, decide (from definition without using any standard series test(s)) whether S_n converges or diverges to ∞.
   (a) Let $0 \leq a \leq 1$ and for $n \geq 1$, let
   $$S_n = \sum_{k=1}^{n} a^k.$$
   (b) Let $p \in \mathbb{R}$ and for $n \geq 1$, let
   $$S_n = \sum_{k=1}^{n} \frac{1}{k^p}$$

2. Suppose $\{z_n\}_{n=1}^{\infty}$ is a sequence of real numbers. Decide whether the series $\sum_{n=1}^{\infty} z_n$ converges in each of the following cases:
   $$\text{(i)} \quad z_n = \frac{n^2 - n + 1}{n^3 + 1}$$
   $$\text{(ii)} \quad z_n = \left(\frac{n}{2n+1}\right)^n$$

3. Suppose $\{z_n\}_{n \geq 1}$ is a sequence of real numbers given by
   $$z_n = \frac{(-1)^n}{\sqrt{n}}.$$
   (a) Decide whether the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.
   (b) Decide whether the series $\sum_{n=1}^{\infty} z_n$ converges.

4. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of positive real numbers.
   (a) Suppose $\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = l$ then show that $\lim_{n\to\infty}(x_n)^{\frac{1}{n}} = l$
   (b) Is the converse of (a) true ?

1. Assume $a_n > 0$ and that $\sum_{n=1}^{\infty} a_n < \infty$. Does it imply that $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}} < \infty$?

2. Consider $\{x_n\}_{n=1}^{\infty}$, such that $x_n = \left(1 + \frac{1}{n}\right)^n$, for all $n \in \mathbb{N}$. Show that x_n is a monotonically increasing sequence and it converges to $x \in \mathbb{R}$.

3. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of non-zero real numbers such that

$$\lim_{n\to\infty} n \left(\left| \frac{x_n}{x_{n+1}} \right| - 1 \right) = L.$$

   (a) Suppose $L > 1$. Show that the series converges absolutely.

   (b) What happens if $L \leq 1$?