1. Let $\phi : G \longrightarrow G$ be a homomorphism. Let $o(x) = n$. What can you say about $o(\phi(x))$?
2. Find all the elements of $G = \mathbb{Z}/12\mathbb{Z}$ which generate G as a cyclic group.
3. List all subgroups of $\mathbb{Z}/48\mathbb{Z}$.
4. Find all cosets of H in S_3, where $H = \langle(12)\rangle$.
5. Prove that $\mathbb{Q}$, the additive group of rational numbers, is not cyclic.
6. Let G be a group of even order. Show that G contains an element of order 2.
7. Prove that the center $Z(G)$ of a group G is a normal subgroup.
8. Prove that every subgroup of index 2 is normal.
9. Prove that a subgroup H of a group G is a normal subgroup if and only if it is the kernel of a homomorphism.
10. Let $\psi : \mathbb{R}^\times \longrightarrow \mathbb{R}^\times$ be the map sending x to the absolute value of x. Prove that ψ is a homomorphism and find the image of ψ. Describe the kernels and the fibres of ψ.
11. Define $\phi : \mathbb{C}^\times \longrightarrow \mathbb{R}^\times$ by $\phi(a + ib) = a^2 + b^2$. Prove that ϕ is a homomorphism and find the image of ϕ. Describe the kernel and the fibres of ϕ geometrically (as subsets of the plane).
12. (**Practice problem**) Consider the additive quotient group $\mathbb{Q}/\mathbb{Z}$.
    (a) Show that every coset of $\mathbb{Z}$ in $\mathbb{Q}$ contains exactly one representative $q \in \mathbb{Q}$ in the range $0 \leq q < 1$.
    (b) Show that every element of $\mathbb{Q}/\mathbb{Z}$ has finite order but that there are elements of arbitrarily large order.
    (c) Show that $\mathbb{Q}/\mathbb{Z}$ is the torsion subgroup of $\mathbb{R}/\mathbb{Z}$.
    (d) Prove that $\mathbb{Q}/\mathbb{Z}$ is isomorphic to the multiplicative group of roots of unity in $\mathbb{C}$.