**June 17th, 2019** **SWMS-2018-Follow up** **Worksheet 2**

**Limit Points- Largest**

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Let the sequence $\{x_n\}_{n=1}^{\infty}$ be a bounded sequence. Let $E$ be the set of limit points of the sequence.

1. $S = \sup\{E\}$ is an element of $E$.

2. Find $S$ in each of the following cases:

   (a) $x_n = \begin{cases} 3 - \frac{1}{n} & \text{if } n = 3k, \text{ for } k \in \mathbb{N} \\ 6 - \frac{1}{n} & \text{if } n = 3k-1, \text{ for } k \in \mathbb{N} \\ 2 - \frac{1}{n} & \text{if } n = 3k-2, \text{ for } k \in \mathbb{N} \end{cases}$

   (b) $x_n = 1 + (-1)^n$

3. Show that for every $\epsilon > 0$

   (a) There exists $N > 0$ such that $x_n < S + \epsilon$ for all $n \geq N$.

   (b) For all $N > 0$ there exists $n > N$ such that $S - \epsilon < x_n$.