Notations:
G stands for a group.
S_n denotes the symmetric group of degree n, ie. the group of permutations on n elements.

1. Let $x \in G$ and let $a, b \in \mathbb{Z}^+$.
   (a) Prove that $x^{a+b} = x^a x^b$ and $(x^a)^b = x^{ab}$.
   (b) Prove that $(x^a)^{-1} = x^{-a}$.
   (c) Establish part (a) for arbitrary integers a and b (positive, negative and zero).
2. Let $G = \{z \in \mathbb{C} \mid z^n = 1 \text{ for some } n \in \mathbb{Z}^+\}$.
   (a) Prove that G is a group under multiplication (called the group of roots of unity in $\mathbb{C}$).
   (b) Prove that G is not a group under addition.
3. For an element $x \in G$ show that x and x^{-1} have the same order.
4. If G is a group such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$, show that G must be abelian.
5. Calculate the order of each element in S_3.
6. If x is an element of infinite order in G, prove that the elements x^n, $n \in \mathbb{Z}$ are all distinct.
7. Compute the orders of all elements in the group $\mathbb{Z}/9\mathbb{Z}$.
8. Assume $G = \{1, a, b, c\}$ is a group of order 4 with identity element 1. Assume also that G has no element of order 4. Find a group multiplication table for G and show that this is unique. Deduce that G is abelian.