June 17th, 2019 **SWMS-2018-Follow up** **Worksheet 1**
Periodicity and Convolution for infinite sequences

Notation: For $a, t \in \mathbb{Z}$

$$\delta_a(t) = \begin{cases} 1 & \text{if } t = a \\ 0 & \text{otherwise} \end{cases}$$

$$u_t = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Determine whether or not each of the following sequences is periodic.
   (a) $x_t = 3\cos(\frac{3\pi}{7}t - \frac{\pi}{8})$
   (b) $x_t = \exp(i(\frac{t}{8} - \pi))$

2. Let $\{a_t\}_{t\in\mathbb{Z}}$ and $\{b_t\}_{t\in\mathbb{Z}}$ be two bi-infinite sequences. Define the convolution c of a and b by

$$c_t = \sum_{s=-\infty}^{\infty} a_s b_{t-s}.$$

   For each of the following cases compute c when

   (a) $a_t = \begin{cases} 1 & \text{if } t = 0, 2 \\ 2 & \text{if } t = 1 \\ 0 & \text{otherwise} \end{cases}$ and $b_t = u_t$ for all $t \in \mathbb{Z}$.

   (b) $a_t = \alpha^t u_t$ and $b_t = \beta^t u_t$ for all $t \in \mathbb{Z}$ with $0 < \alpha, \beta < 1$.

3. Let $a_t = u_t$, $b_t = \delta_0(t) - \delta_{-3}(t)$, $c_t = (0.8)^t u_t$ for all $t \in \mathbb{Z}$. Verify $(a_t * b_t) * c_t = a_t * (b_t * c_t)$