1. Let V be the space of all 6×6 real matrices. Find the dimension of the subspace of V consisting of all symmetric matrices.

2. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be the linear transformation defined by $T(x, y, z) = (x+2y, x-z)$. Let $n(T)$ be the null space of T and $W = \{v \in \mathbb{R}^3 | v \cdot u = 0, \forall u \in n(T)\}$. Find a linear transformation $S : \mathbb{R}^2 \longrightarrow W$ such that $TS = I$, where I is the identity transformation of $\mathbb{R}^2$.

3. Let A be a real square matrix of odd order such that $A + A^T = 0$. Prove that A is singular.

4. Let A be an $n \times n$ matrix such that $A^n = 0$ and $A^{n-1} \neq 0$. Show that there exists a vector $v \in \mathbb{R}^n$ such that $\{v, Av, \ldots, A^{n-1}v\}$ forms a basis for $\mathbb{R}^n$.

5. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation such that $T((1, 2)) = (2, 3)$ and $T((0, 1)) = (1, 4)$. Find the value of $T((5, 6))$.

6. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear transformation whose matrix with respect to the unit vectors e_1, e_2, e_3 is
$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$
Then, which of the following is (are) true:
(a) T maps the subspace spanned by e_1 and e_2 into itself.
(b) T has distinct eigenvalues.
(c) T has eigenvectors that span $\mathbb{R}^3$.
(d) T has a non-zero null=space.

7. (a) Give an example of a linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that $T^2(v) = -v$ for all $v \in \mathbb{R}^2$.
(b) Let V be a real n dimensional vector space and let $T : V \longrightarrow V$ be a linear transformation satisfying $T^2(v) = -v$ for all $v \in V$. Show that n is even.

8. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a linear transformation, where $n \geq 2$. For $k \neq n$ let $E = \{v_1, v_2, \ldots, v_k\} \subseteq \mathbb{R}^n$ and let $F = \{Tv_1, Tv_2, \ldots, Tv_k\}$.
Then which of the following is (are) true:
(A) If E is linearly independent, then F is linearly independent.
(B) If F is linearly independent, then E is linearly independent.
(C) If E is linearly independent, then F is linearly dependent.
(D) If F is linearly independent, then E is linearly dependent.

9. For $n \neq m$, let $T_1 : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $T_2 : \mathbb{R}^m \longrightarrow \mathbb{R}^n$ be linear transformations such that T_1T_2 is bijective, Then, which of the following is (are) true:

(i) rank(T_1)= n and rank(T_2)=m.
(ii) rank(T_1)= m and rank(T_2)=n.
(iii) rank(T_1)= n and rank(T_2)=n.
(iv) rank(T_1)= m and rank(T_2)=m.

10. Let $\{v_1, v_2, v_3\}$ be a basis of a vector space V over$\mathbb{R}$. Let $T : V \longrightarrow V$ be the linear transformation determined by: $T(v_1) = v_1$, $T(v_2) = v_2 - v_3$ and $T(v_3) = v_2 + 2v_3$. Find the matrix of the transformation T with $\{v_1 + v_2, v_1 - v_2, v_3\}$ as a basis of both the domain and the co-domain of T

11. Consider the subspace $W = \{(x_1, x_2, \ldots, x_{10}) \in \mathbb{R}^{10} : x_n = x_{n-1} + x_{n-2}$ for $2 \leq n \leq 10\}$ of the vector space $\mathbb{R}^{10}$. Find the dimension of W.

12. Let $B_1 = \{(1, 2), (2, -1)\}$ and $B_2 = \{(1, 0), (0, 1)\}$ be ordered bases of $\mathbb{R}^2$. If $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation such that $[T]_{B_1,B_2}$, the matrix of T with respect to $B_!$ and B_2, is

$$\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$$

then $T(5, 5)$ is equal to
(A) $(-9, 8)$ (B) $(9, 8)$ (C) $(-15, -2)$ (D) $(15, 2)$.