1. Suppose N is a normal subgroup of a group G. Which one of the following is true?
(A) If G is an infinite group then G/N is an infinite group.
(B) If G is a nonabelian group then G/N is a nonabelian group.
(C) If G is a cyclic group then G/N is an abelian group.
(D) If G is an abelian group then G/N is a cyclic group.

2. Which of the following statements is (are) true?
(A) $\mathbb{Z}_2 \times \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$.
(B) $\mathbb{Z}_3 \times \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_9$.
(C) $\mathbb{Z}_4 \times \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$.
(D) $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{30}$.

3. Find the number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$.

4. Let G be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?
(A) G is abelian if $o(G) = pq$ where p and q are distinct primes.
(B) G is abelian if every non identity element of G is of order 2.
(C) G is abelian if the quotient group $G/Z(G)$ is cyclic, where $Z(G)$ is the center of G
(D) G is abelian if $o(G) = p^3$, where p is prime.

5. In the permutation group $S_n, (n \geq 5)$, if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?
(A) Order of H is 2.
(B) Index of H in S_n is 2.
(C) H is abelian.
(D) $H = S_n$.

6. Let H denote the group of all 2×2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication. Then find the order of the matrix
$$a = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

7. Let G be a group of order 20 where the conjugacy classes have sizes $1, 4, 5, 5, 5$. Then which of the following is/are true.
(a) G contains a normal subgroup of order 5.
(b) G contains a non-normal subgroup of order 5.
(c) G contains a subgroup of order 10.
(d) G contains a normal subgroup of order 4.

8. Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.
I. Every cyclic subgroup of H is finite.
II. Every finite cyclic group is isomorphic to a subgroup of H.
Which one of the following holds?
(A) I is TRUE but II is FALSE
(B) II is TRUE but I is FALSE
(C) both I and II are TRUE
(D) neither I nor II is TRUE.

9. Consider the group $\mathbb{Z} \times \mathbb{Z} = \{(a, b) | a, b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z} \times \mathbb{Z}$?
(A) $\{(a, b) \in \mathbb{Z} \times \mathbb{Z} : ab = 0\}$.
(B) $\{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 3a + 2b = 15\}$.
(C) $\{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 7 \text{ divides } ab\}$.
(D) $\{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$.

10. Let A_6 be the group of even permutations of 6 distinct symbols. Then find the number of elements of order 6 in A_6.