Discrete Time Fourier Transform for infinite sequences- Filters

1. Show that
$$\sum_{t=-\infty}^{\infty} a * b_t e^{-i2\pi ft} = A(f)B(f)$$

2. (High Pass Filter) Suppose
$$a_t = \begin{cases} 1/2, & \text{t=0;} \\ -1/4, & t = \pm 1; \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } b_t = \frac{3}{16}\left(\frac{4}{5}\right)^{|t|} + \frac{1}{20}\left(-\frac{4}{5}\right)^{|t|}, t \in \mathbb{Z}$$
   (a) Find $A(f)$ and $B(f)$
   (b) Plot $A(f)B(f)$.

3. (Low Pass Filter) Suppose
$$a_t = \begin{cases} 1/2, & \text{t=0;} \\ 1/4, & t = \pm 1; \\ 0, & \text{otherwise.} \end{cases} \quad \text{and } b_t = \frac{3}{16}\left(\frac{4}{5}\right)^{|t|} + \frac{1}{20}\left(-\frac{4}{5}\right)^{|t|}, t \in \mathbb{Z}$$
   (a) Find $A(f)$ and $B(f)$
   (b) Plot $A(f)B(f)$.

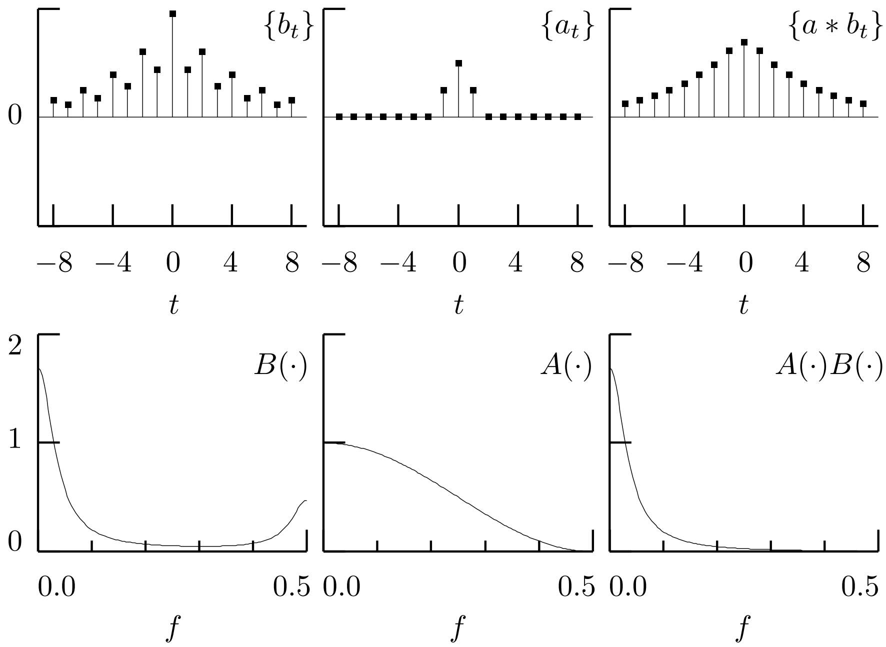

Figure 26. Example of filtering using a low-pass filter.

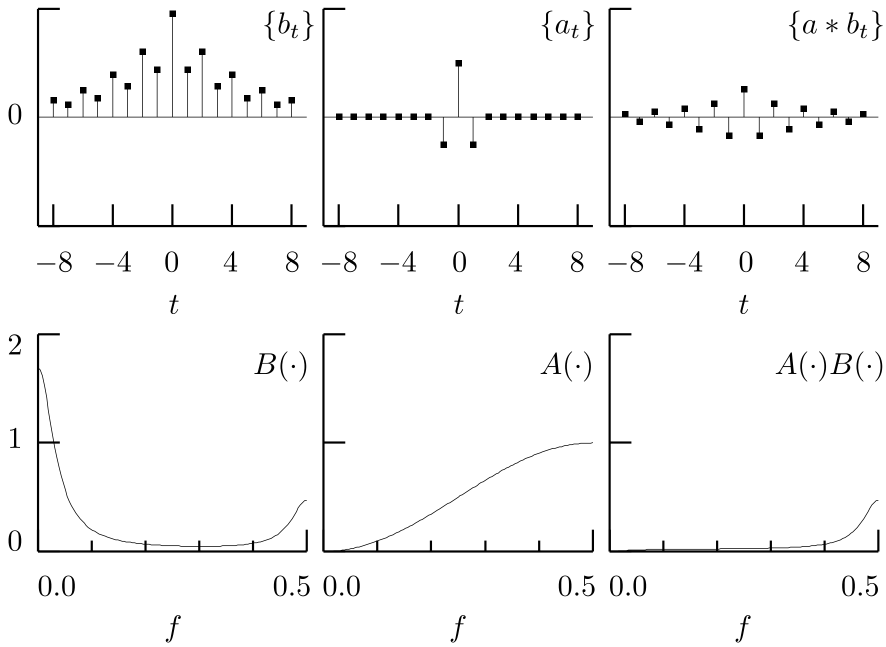

Figure 27. Example of filtering using a high-pass filter.