1. Let A be a non-empty set of real numbers which is bounded below. Let $-A := \{x \in \mathbb{R} : -x \in A\}$. Show that $\inf(A) = -\sup(-A)$.

2. If $z, w, z_i \in \mathbb{C}$ for $i = 1, 2, \ldots, n$ then show that

$$|z_1 + z_2 + \cdots + z_n| \leq |z_1| + |z_2| + \cdots + |z_n|$$

and

$$||z| - |w|| \leq |z - w|$$

3. Let A and B be bounded nonempty subsets of $\mathbb{R}$, and let $A + B := \{a + b : a \in A, b \in B\}$. Prove that $\sup(A + B) = \sup(A) + \sup(B)$.

4. In each of the cases below decide if $\{x_n\}_{n=1}^{\infty}$either converges to a real number or diverges to ∞ or diverges to $-\infty$ or none of the above.

   (a) $x_n = \frac{2n^2+1}{3n^2-1}$

   (b) $x_n = n^{(-1)^n}$

   (c) $x_n = \frac{n!}{n^n}$

June 3rd, 2019 **SWMS-2018-Follow up**

Practice Problems Set 3

1. Decide if $\{x_n\}_{n=1}^{\infty}$either converges to a real number or diverges to ∞ or diverges to $-\infty$ or none of the above when $x_n = (a_1^n + a_2^n + a_3^n)^{\frac{1}{n}}$, with $a_1, a_2, a_3 > 0$

2. Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence. Then it has a subsequence convergent in $\mathbb{R}$. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers.

3. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers $\mathbb{R}$ and suppose that $x_n \to x$.

   (a) Let $m, n \in \mathbb{N}$, show that $x_{m+n} \to x$ as $m \to \infty$.

   (b) Let $m, l \in \mathbb{N}$, $p : \mathbb{R} \to \mathbb{R}$ such that $p(x) = \sum_{k=0}^{l} p_k x^k$, and $q : \mathbb{R} \to \mathbb{R} \backslash \{0\}$, $q(x) = \sum_{k=0}^{m} q_k x^k$, with $p_k \in \mathbb{R}, q_k \in \mathbb{R}$ for $k = 1, 2, \ldots, n$. Show that if $r : \mathbb{R} \to \mathbb{R}$ defined by $r(x) = \frac{p(x)}{q(x)}$ then $r(x_n) \to r(x)$.

   (c) Show that $\{\mid x_n \mid\}_{n=1}^{\infty}$ also converges