1. In each of the cases below decide if $\{x_n\}_{n=1}^{\infty}$converges or not:
   (a) $x_n = \frac{1}{n}$
   (b) $x_n = \sqrt{n^2 - n} - n$
   (c) $x_n = \frac{2^n}{n!}$,
   (d) $x_n = nb^n$, for $b \in (0, 1)$
2. Let a_n be a bounded sequence of real numbers. Let $s = \sup\{a_n : n \geq 1\}$ and $s \notin \{a_n : n \in \mathbb{N}\}$. Show that there is a subsequence of a_n that increases to s.
3. Suppose $\{a_n\}_{n\geq 1}$ is a sequence of real numbers such that
$$\lim_{n\to\infty} \mid a_{n+1} - a_n \mid = 0.$$
Does this necessarily imply that the sequence is a Cauchy sequence.
4. Consider the $\{y_n\}_{n=1}^{\infty}$, such that $y_1 > 1$ and $y_{n+1} := 2 - \frac{1}{y_n}$ for $n \geq 2$. Show that y_n converges.
5. (*Finding Roots of a number*) Let $a > 0$ and choose $s_1 > \sqrt{a}$. Define
$$s_{n+1} := \frac{1}{2}(s_n + \frac{a}{s_n})$$
for $n \in \mathbb{N}$.
   (a) Show that s_n is monotonically decreasing and $\lim_{n\to} s_n = \sqrt{a}$.
   (b) If $z_n = s_n - \sqrt{a}$ then show that $z_{n+1} < \frac{z_n^2}{2\sqrt{a}}$.
   (c) Let $f(x) = x^2 - a$. Show that $s_n = s_{n-1} - \frac{f(s_{n-1})}{f'(s_{n-1})}$.
   (d) Draw graph of f with $a = 4$ and plot the sequence s_n for a few steps when $s_0 = 5$.

June 3rd, 2019 **SWMS-2018-Follow up**

Practice Problems Set 2

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1. Decide if $\{x_n\}_{n=1}^{\infty}$either converges to a real number or diverges to ∞ or diverges to $-\infty$ or none of the above when $x_n = n^{\frac{1}{n^2}}$

2. Let $x \in \mathbb{R}$, $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers. Show that the following are equivalent:

   (a) $\forall \epsilon > 0$, there is an $N \in \mathbb{N}$ such that $\mid x_n - x \mid < \epsilon$ for all $n \geq N$.

   (b) $\forall \epsilon \in (0, 1)$, there is an $N \in \mathbb{N}$ such that $\mid x_n - x \mid < \epsilon$ for all $n \geq N$.

   (c) Let $C > 0$, $\forall \epsilon > 0$, there is an $N \in \mathbb{N}$ such that $\mid x_n - x \mid \leq C\epsilon$ for all $n > N$.