1. Provide two examples (if any) of sequences $\{a_n\}$ that satisfy each of the statements below.
   (a) A sequence that converges to 0 which has the property that infinitely many elements are negative numbers and infinitely many elements are positive numbers.
   (b) For all $\epsilon > 0$, there are infinitely many $n \in \mathbb{N}$ such that
   $$| a_n - L | < \epsilon$$
   for $L = -1, 0, 3$ and $a_n \notin \{-1, 0, 3\}$ for infinitely many $n \in \mathbb{N}$.
   (c) A sequence that converges to 0 which has the property that all but finitely many elements are negative.
   (d) For all $\epsilon > 0$, for all but finitely many $n \in \mathbb{N}$ $a_n < 5 + \epsilon$ and $a_n > -11 - \epsilon$.
   (e) For all $M > 0$:
   there are infinitely many $n \in \mathbb{N}$ such that $a_n > M$ and
   there are infinitely many $n \in \mathbb{N}$ such that $a_n < -M$.

   Definition: A $L \in \mathbb{R}$ is a limit point of $\{x_n\}_{n=1}^{\infty}$if for every $\epsilon > 0$, $(L - \epsilon, L + \epsilon)$ has infintely many elements of the sequence in it.

2. Provide an example (if any) of sequences $\{a_n\}$ that satisfy each of the statements below. If there is no example then please justify.
   (a) A sequence that has limit points $L = 0$ and $L = 5$.
   (b) A sequence that converges to 2 and 3 is a limit point.

3. Find the limit points of the $\{x_n\}_{n=1}^{\infty}$in each of the following cases:
   (a) $x_n = \begin{cases} \frac{n}{n+1} & \text{if } n \text{ is odd} \\ \frac{1}{n+1} & \text{if } n \text{ is even} \end{cases}$
   (b) $x_n = \begin{cases} 2^{\frac{1}{n+1}} & \text{if } n \text{ is odd} \\ \frac{1}{n+1} & \text{if 1 is even} \end{cases}$
   (c) $x_n = 1 + (-1)^n$
   (d) $x_n = (1 + \frac{(-1)^n}{n})^n$

   In each of the above list the largest and the smallest limit points.

4. Let L be a limit point of a sequence $\{x_n\}_{n=1}^{\infty}$. Show that there is a subsequence of $\{x_{n_k}\}$ that converges to L.

5. Suppose $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers and let E its set of limit points. Decide if the following Show that $E = \{x\}$ for some $x \in \mathbb{R}$ if and only if $\{x_n\}_{n=1}^{\infty}$converges to x.