1. Let $A \subset \mathbb{R}$. Define:
   (a) What is meant by saying A is a bounded subset of $\mathbb{R}$?
   (b) Negate the above statement using logical notation.
   (c) What is meant by saying $\alpha = \sup(A)$ and $\beta = \inf(A)$?

2. Find the infimum and supremum of the sets
   (a) $B = \{2, 3, 4\}$
   (b) $S = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{50 + \frac{1}{n} : n \in \mathbb{N}\}$
   (c) $A = \{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\}$

3. Let $A = \{p \in \mathbb{Q} : p^2 < 2\}$. Show that A is bounded above but does not have a least upper bound in $\mathbb{Q}$

4. Let A and B be two subsets of $\mathbb{R}$, which are both bounded below. Let $u = \inf(A)$ and $v = \inf(B)$. Find $\inf(C)$, in terms of u, v, when
   (a) $C = \{a + b : a \in A, b \in B\}$,
   (b) $C = A \cup B$,
   (c) $C = A \cup \{10\}$.

5. Let $n \geq 1$ and $x_i > 0$. Prove the Bernoulli inequality:

$$\prod_{i=1}^{n}(1 + x_i) \geq 1 + \sum_{i=1}^{n} x_i$$

6. If $a \in \mathbb{R}$ such that $0 \leq a < \epsilon$ for every $\epsilon > 0$, then show that $a = 0$.

June 3rd, 2019 **SWMS-2018-Follow up**

Practice Problems Set 1

1. Let A and B be two subsets of $\mathbb{R}$, which are both bounded below. Let $u = \sup(A)$ and $v = \sup(B)$. Find $\sup(C)$, in terms of u, v, when
   (a) $C = \{a + b : a \in A, b \in B\}$,
   (b) $C = A \cup B$, (What if $C = A \cap B$?)
   (c) $C = A \cup \{-10\}$.
2. Let A and B be two non-empty subsets of $\mathbb{R}$. Define what is meant by a function $f : A \to B$ and when is it called one-one and onto. Provide examples of f that are (and are not) one-one and (or) onto when A, B are finite or countable or uncountable.