June 13th, 2019 **SWMS-2018-Follow up**
Worksheet 1 **Discrete Fourier Transform**

Let $\{a_t : t = 0, 1, 2, \ldots, N-1\}$ be a set of real(or complex) numbers (sometimes called finite sequence). Define its discrete Fourier transform (DFT) to be the sequence

$$A_k = \sum_{t=0}^{N-1} a_t e^{-i\frac{2\pi tk}{N}}, \; k = 0, \ldots, N-1. \tag{1}$$

1. Suppose we extend the definition of A_k above for all $k \in \mathbb{Z}$ using the same formula as in (2). Show that for any non-zero integer $n \in \mathbb{Z}$ and any integer k such that $0 \leq k \leq N-1$,

$$A_{k+nN} = A_k.$$

   That is, *the resulting infinite sequence if periodic with period of N.*

2. Show that

$$\frac{1}{N} \sum_{k=0}^{N-1} A_k e^{i\frac{2\pi tk}{N}} = a_t, \; t = 0, 1, \ldots, N-1.$$

3. Compute a when its DFT is given by
   (a) $N = 2$, $A_0 = 0, A_1 = 1$.
   (b) $N = 2$, $A_0 = 0, A_1 = 1, A_2 = 1$.

4. Compute DFT of a when it is given by
   (a) $N = 4$, $a_0 = 4, a_1 = 0, a_2 = 0, a_3 = 0$.
   (b) $N = 4$, $a_0 = 0, a_1 = 4, a_2 = 0, a_3 = 0$.

5. From the above can you provide a physical explanation for the fourier transform.