Recall:-

Existence ?:-
construction

$\mathbb{N}$ – Peano's axiom
$\mathbb{Z}$ – integers, $+$
$\mathbb{Q}$ – rationals $\times$
$\mathbb{R}$ – Real numbers

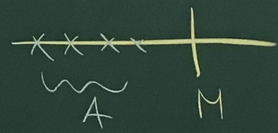
• A = Bounded subset in $\mathbb{R}$

$$\exists M > 0 \quad \forall x \in A \quad |x| \le M$$

• **Bounded above**

$\exists M \in \mathbb{R}$ such that

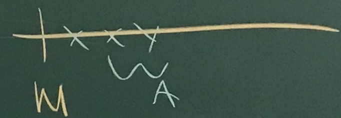
$$x \le M$$



• $\alpha = \mathrm{Sup}(A)$ – least upper bound
  • α is an upper bound of A
  • $\beta < \alpha$ then β is NOT an upper bound

Bounded below

$\exists M \in \mathbb{R}$ such that
$\forall x \in R \quad x > M$



• $\gamma = \inf(A)$
  – γ was a lower bound of A
  – $\eta > \gamma$, η is not a lower bound of A

Se

• a_n

$\forall \varepsilon >$

all but
finitely

• a^n –
a_n n

• $n b^n$

• $\frac{2^n}{n!}$

Sequences $\{a_n\}_{n\geq 1}$ $(a : \mathbb{N} \to \mathbb{R})$

- $a_n \to a$ as $n \to \infty$ if

$\forall \varepsilon > 0 \ \exists N > 0 : \ \forall n \geq N, |a_n - a| < \varepsilon$

$a_n \in$ if $n \geq N$

a_{N-1} $a-\varepsilon$ a $a+\varepsilon$

all but finitely many are in any arbitrarily small interval around a

- $a^n \to 0$ as $n \to \infty$ $\quad 0 < a < 1$
- $\frac{1}{n} \to 0$ as $n \to \infty$

$(\varepsilon > 0 \ \exists n_0 : n_0 > \frac{1}{\varepsilon})$

- $n b^n \to 0$ as $n \to \infty$
- $\frac{2^n}{n!} \to 0$ as $n \to \infty$

Hints W1 - Problem 3

In order to show that A does not have

$t_0^2 < 2, t_0$ $\quad s_0 < s$ & $s_0^2 > 2$

t $\quad 2 \quad s$

$t^2 < 2$ $\quad \boxed{s^2 > 2}$

There does not exist $r \in \mathbb{Q}$ s.t. $r^2 = 2$

$\mathbb{R}$ — has this Property

"any set bounded above has a least upper bound"

$\Updownarrow$

any Cauchy sequence in $\mathbb{R}$ converges. | any convergent sequence is Cauchy