

2⊗ Suppose $\exists \in E \cap F$

$\Rightarrow S \in E \text{ \& } S \in F$

$\Rightarrow [S \in R_1 \text{ or } S \in R_2] \text{ \& } S \in R_1^c \text{ \& } S \in R_2$

If $S \in R_1$ then we know $S \in R_1^c$

$$[R_1 \cap R_2^c] \cup [G_1 \cap G_2^c] \cup [(G_1 \cup R_1) \cap (G_2 \cup R_2)]$$

$$2\textcircled{c} R_1 \cap R_2^c \cup G_1 \cap G_2^c \cup W_1 \cap W_2^c$$

$$A = A \cap B \cup A \cap B^c \quad | \quad A \cup B = A \cap B^c \cup B \cap A^c \cup A \cap B$$

$$B = B \cap A \cup B \cap A^c$$

we have a

So, contradiction ($\because R_1 \cap R_1^c = \emptyset$)

If $S \in G_2$ then we know $S \in R_2$, So, contradiction ($\because G_2 \cap R_2 = \emptyset$)

$$E \cap F = \emptyset \quad \square$$

$$3\textcircled{c} A \cap B = A \cup B \setminus A \cap B$$



Rules

Required

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (P(A \cap B) + P(A \cap B^c)) + (P(B \cap A) + P(B \cap A^c)) - P(A \cap B)$$



FROM AXIOMS

① $P(\phi) = 0$

[Claim 2]

② $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

if $E_1 \cap E_2 = \phi$

[Claim 1]

* [Induction] $P(\bigcup_{i=1}^n E_i) = \sum_{i=1}^n P(E_i)$ whenever $E_i \cap E_j = \phi$ ($i \neq j$)

Proof of A

$n=2$ verified in ②

Assume $n=k$
* for

$n=k+1$ $E_1, \dots, E_{k+1}$ s.t.
 $E_i \cap E_j = \phi$ ($i \neq j$) [$\bigcup_{i=1}^k E_i \cap E_{k+1} = \phi$]

$P(\bigcup_{i=1}^{k+1} E_i) = P(\bigcup_{i=1}^k E_i \cup E_{k+1})$

$\stackrel{n=2}{=} P(\bigcup_{i=1}^k E_i) + P(E_{k+1})$
 $\stackrel{n=k}{=} \sum_{i=1}^k P(E_i) + P(E_{k+1}) = \sum_{i=1}^{k+1} P(E_i)$

$P(A) = 0.7$

$P(B) = 0.6$

$P(A \cup B) = 0.9$

$P(A \cap B) = 0.7 + 0.6 - 0.9 = 0.4$

$P(A \cap B^c) = P(A) - P(A \cap B) = 0.3$

[$A = A \cap B \cup A \cap B^c$]

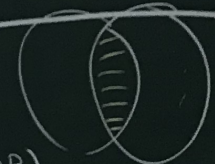
$P(B \cap A^c) = P(B) - P(A \cap B) = 0.2$

$P(A \cup B) = P(A \cap B^c) + P(B \cap A^c) + P(A \cap B)$
 $= 0.3 + 0.2 + 0.4 = 0.9$

Rest of Table
Ex.

③

$A \cap B = A \cup B \setminus (A \cap B^c \cup A^c \cap B)$



Rules

Required

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= (P(A \cap B) + P(A \cap B^c)) + (P(A^c \cap B) + P(A \cap B)) - P(A \cap B)$



CONDITIONAL PROBABILITY

B = 1st roll of 2

$$P(E|B) = \frac{1/36}{6/36} = 1/6$$

$$P(E|B) = P(\{2,4\}) = \frac{1}{36}$$

$$P(B) = P(\{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}) = 6/36$$

Question 4

$$P(E|B) = \frac{\#E \cap B}{\#B} = \frac{\#E \cap B}{\#B}$$

- S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

$$\mathcal{F} = \{ E \mid E \subseteq S \}$$

Uniform (S)

$$|S| < \infty$$

Equally likely outcome

$$P: \mathcal{F} \rightarrow [0,1]$$

$$P(E) = \frac{\#E}{\#S}$$

E = Sum of 2 rolls of 2
= { (4,0), (3,3), (4,4), (5,3), (6,0) }

$$P(E) = 5/36$$

$$P(E) \in [0,1]$$

P-satisfying Axioms:

$$(1) P(S) = 1$$

$$(2) \#E_1 \cup E_2 = \#E_1 + \#E_2 \text{ if } E_1 \cap E_2 = \emptyset$$

$$\lim_{L \rightarrow \infty} \frac{\# \bigcup_{i=1}^L E_i}{\#S} = \sum_{i=1}^{\infty} \frac{\#E_i}{\#S}$$

$$\text{Induction } \frac{\# \bigcup_{i=1}^k E_i}{\#S} = \sum_{i=1}^k \frac{\#E_i}{\#S}$$