June 8th, 2019 **SWMS-2018-Follow up**
Weekend **Homework**

Due on June 10th, 2019: 1, 2,3,5,6, 8(a) [Not 5], 8(c) only 2.

1. Decide if in the following $\{x_n\}_{n=1}^{\infty}$converges to a real number or diverges to $pm\infty$ or neither.
   (a) $x_n = n^{\frac{1}{n}}$
   (b) $x_n = \frac{n^4+1}{-4n^4+2n^5}$

2. Practice Problem Set 1 June 3rd, 2019: 1.
3. Worksheet 1 June 6th, 2019: 4 (a) and 4(b).
4. Hw : June 6th, 2019 : 4,5.
5. Decide if in the following $\sum_{n=1}^{\infty} x_n$ converges to a real number.
   (a) $x_n = 1 - 2^{\frac{1}{n}}$
   (b) $x_n = \frac{n^4+1}{(n^2+2)^2}$

6. Decide if in the following $\sum_{n=1}^{\infty} x_n$ converges and converges absolute.
   (a) $x_n = (-1)^n b^n$
   (b) $x_n = (-1)^n \frac{1}{n^p}$ with $p \in \mathbb{R}$.
   (c) $x_n = (-1)^n \frac{2^n}{n!}$

7. Suppose $\{y_n\}_{n=1}^{\infty}$is such that $\sum_{n=1}^{\infty} y_n = t$ with $t \in \mathbb{R}$.
   (a) Let $\mathrm{T}_n = t - \sum_{k=1}^{n} y_k$. Does T_n have a limit as $n \to \infty$?
   (b) Does y_n have a limit as $n \to \infty$?

8. (Not Due) Backlog List:-
   (a) Worksheet 1 (June 3rd, 2019): 2(c),3, 4(b), 4(c), 5,6.
   (b) Practice Problem Set 1 (June 3rd, 2019): **1**, 2.
   (c) Worksheet 2 (June 3rd, 2019): **2**,4, 5b, 5c, 5d
   (d) Practice Problem Set 2 (June 3rd, 2019): 2.
   (e) Practice Problem Set 1 (June 6th, 2019): 1,2,3.
   (f) Worksheet 3 (June 6th, 2019): 1e, 1f, 1g.

1. Let G be an abelian group and let N be a normal subgroup of G. Then prove that the quotient group G/N is also an abelian group.
2. Let $a, b \in G$. Show that both the elements ab and ba have the same order.
3. Let G, G' be groups. Suppose that we have a surjective group homomorphism $\phi : G \to G'$. Show that if G is an abelian group, then so is G'.
4. Fins the automorphism group of $\mathbb{Z}/3\mathbb{Z}$.
5. Find the automorphism group of
   (a) $\mathbb{Z}/4\mathbb{Z}$ and
   (b) the Kleins' 4-group.
6. What are all the homomorphisms from Q_8, the group of quaternions, to $\mathbb{C}^\times$, the multiplicative group of all nonzero complex numbers.
7. Let G be a finite group. Suppose that $Aut(G) = \{Id\}$. Show that the group G is abelian.
8. Use first isomorphism theorem to prove that $GL_n(\mathbb{R})/SL_n(\mathbb{R}) \cong \mathbb{R}^\times$, where $\mathbb{R}^\times$ is the multiplicative group of nonzero real numbers.
9. Prove that the quotient group of a cyclic group is cyclic.
10. Suppose N is normal of G. Show that for a subgroup H of G, $H \cap N$ is a normal subgroup of H.