1. Check for solvability of the following system of linear equations, and if solvable, find a solution:

$$\begin{array}{rrrl} x & +y & -z = & 3 \\ x & -3y & +2z = & 1 \\ 2x & -2y & +z = & 4. \end{array}$$

2. Prove that a system of m homogeneous equations in n unknowns, where $n > m$, always has a non-trivial solution.

3. Find a basis of the solution space of the system

$$\begin{array}{lllll} 3x_1 & -x_2 & & +x_4 & = 0 \\ x_1 & +x_2 & +x_3 & +x_4 & = 0 \end{array}$$

4. Let $N \geq 1$. Consider the $N \times N$ matrix $A = (a_{kl})$ whose entries are given by

$$a_{kl} = \frac{1}{\sqrt{N}} \exp(-\frac{i2\pi kl}{N})$$

for $1 \leq k, l \leq N$. Decide if the matrix is orthogonal. If so then describe its inverse.