1. Decide (from definition) whether the following sequence $\{y_n\}_{n=1}^{\infty}$ is Cauchy or not:

$$y_n = \sum_{k=1}^{n} \frac{1}{k!}$$

2. Suppose $\{z_n\}_{n\geq 1}$ is a sequence of real numbers. Without using the ratio or root test decide whether the series $\sum_{n=1}^{\infty} z_n$ converges when

$$z_n = \frac{\sqrt{n}}{2n^3 - 1}$$

3. Let $\{a_n\}_{n\geq 1}$ be a sequence of non-negative numbers and $\{a_{n_k}\}_{k\geq 1}$ be a subsequence of the same. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Does it imply that $\sum_{k=1}^{\infty} a_{n_k}$ converges?

4. Let $p \in \mathbb{R}$. Decide whether $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{(n+1)^p}$ converges.

5. Let $x \in \mathbb{R}$. Decide the range of $x \in \mathbb{R}$ where $\sum_{n=1}^{\infty} \frac{(4x-12)^n}{(-3)^{2+n}(n^2+1)}$ converges and diverges.