1. Let G be a finite group of order 21 and let K be a finite group of order 49. Suppose that G does not have a normal subgroup of order 3. Then determine all group homomorphisms from G to K.

2. Let $\mathbb{R} = (\mathbb{R}, +)$ be the additive group of real numbers and let $\mathbb{R}^\times$ be the multiplicative group of non-zero real numbers.
(a) Prove that the map $exp : \mathbb{R} \longrightarrow \mathbb{R}^\times$ defined by $exp(x) = e^x$ is an injective group homomorphism.
(b) Prove that the additive group $\mathbb{R}$ is isomorphic to the multiplicative group of positive real numbers. What is the inverse homomorphism?

3. Show that a group G is cyclic if and only if there exists a surjective group homomorphism from the additive group $\mathbb{Z}$ of integers to the group G.

4. Consider S_3, the symmetric group on 3 elements.
   (i) Find all the automorphisms $\phi : S_3 \longrightarrow S_3$.
   (ii) Find all subgroups of S_3. Which ones are cyclic?
   (iii) Find all normal subgroups.