1. *(Reverse Odd town)* Let $X = \{1, 2, ..., n\}$, and $A_1, A_2..., A_m$ be subsets of X with the property that for all i, $|A_i| =$ even, and $|A_i \cap A_j| =$ odd, for all $i \neq j$. Prove that $m \leq n$ (as usual $|A|$ denotes cardinality of the set A).

2. Let X be as in problem 1, and $A_1, A_2..., A_m$ be subsets of X with the property that $|A_i \cap A_j| = k$, for all $i \neq j$, where $1 \leq k < n$. Let v_i be the incidence vector of A_i. Show directly from the definition that $\{v_1, v_2..., v_m\}$ is a linearly independent set over $\mathbb{R}$, hence giving another proof that $m \leq n$.

3. Prove that in *any* coloring of the edges of K_6 by red or blue, one can always find a red K_3, or a blue K_3. Prove that K_6 cannot be replaced by K_5. You have just shown that $R(3,3) = 6$.

4.Demonstrate the inequality that $R(t,t) > (t-1)^2$ by explicilty constructing a two-coloring.

5. Fix a $z \in \mathbb{C}$. Consider $x_t = z^t$ for all $t \in \mathbb{Z}$.

   (a) Show that for any sequence $\{a_t\}_{t \in \mathbb{Z}}$, we have

   $$a_t * x_t = \sum_{s \in \mathbb{Z}} a_s x_{t-s} = c x_t$$

   for all $t \in \mathbb{Z}$ with c being a constant. What would you call x to be ?

   (b) If $\tilde{x}_t = z^t u_t$ for all $t \in \mathbb{Z}$ then does $\tilde{x}$ have the same property that x has ?