21 days. Why? Make it permanent

$\{a \cdot b \mid a \in A, b \in B\}$

$u = \inf(A)$ $\quad v = \inf(B)$

Claim: $\inf(C) = u \cdot v$

Case 1: $u = 0$ or $v = 0$ $\boxed{Ex?}$

2: $u > 0$ & $v > 0$

let z be another lower bound.

$ab \geq z \quad \forall a, b \in A$

Note $a > 0$ and $b > 0$ (why?)

$\Rightarrow b \geq z/a \quad \forall b \in B$

[v is g.l.b] $\Rightarrow v \geq z/a$ (why?)

[u is

$\Rightarrow a \geq z/v \quad \Rightarrow u \geq z/v$

$\forall a \in A \quad \Rightarrow vu \geq z$

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(2.) Suppose not bounded above.

$\forall M > 0 \quad \exists \; a_N$ such that $a_N > M$ $(*)$ → $(*t)$

(✗ bounded above)

$M = 1 \quad \exists \; a_{n_1}$ such that $a_{n_1} > 1$ ✓

Q:- Find $n_2 > n_1$ & $a_{n_2} > 2$ [?] ← [?]

$$M_2 = \max\{\underbrace{\max\{a_1, \ldots, a_{n_1}\} + 1}_{n_2 > n_1}, 2\}$$

$a_{n_2} \geq 2$

$(**)$ $\exists\, n_2 > n_1 : a_{n_2} > M_2$

$a_{n_2} \geq 2$

$\exists \; n_3 > n_2 \qquad a_{n_3} \geq 3$!(Ex)

induction $\quad n_1 < n_2 < n_3 \ldots < n_k < \cdots$

$a_{n_k} > k \qquad \{a_{n_k}\}_{k \geq 1}$ subsequence

claim: $a_{n_k} \to \infty$ (Ex!)

$p > 0$

$\alpha < 0$

$\boxed{-\alpha > 0}$

$$\frac{n^{\alpha}}{(1+p)^n} = \frac{1}{n^{-\alpha}} \cdot \frac{1}{(1+p)^n} \longrightarrow 0 \text{ as } n \to \infty \checkmark$$

as $n \to \infty$ 0 $\qquad$ 0 as $n \to \infty$

Directly :- $\qquad 0 < \dfrac{n^{\alpha}}{(1+p)^n} < \dfrac{1}{(1+p)^n}$

$\alpha \leq 0$

$\dfrac{n^{\alpha}}{(1+p)^n}$ & $\alpha > 0 \qquad k = \lfloor \alpha \rfloor + 1$

$n > 2k \qquad (1+p)^n = \sum_{r=0}^{n} {}^{n}C_r\, p^r$

$> {}^{n}C_k\, p^k$

$$\frac{n^{\alpha}}{(1+p)^n} < \frac{n^{\alpha}}{\underbrace{{}^{n}C_k}_{\sim\, n^{\lfloor \alpha \rfloor + 1}} p^k} \longrightarrow 0 \text{ as } n \to \infty$$

Ex