1. Let $X = \{1, 2, ..., n\}$. Suppose there are two families of subsets $A_1, A_2.., A_m$ and $B_1, B_2..., B_m$ of X with the property that $|A_i \cap B_i|$ is odd and $|A_i \cap B_j|$ is even for all $i \neq j$. Prove that $m \leq n$.

2. Prove that if A is a symmetric matrix, then $rk(A) \geq \dfrac{(tr(A))^2}{||A||_F^2}$. As before $||A||_F^2$ is the Frobenius norm of A which is given by $\sum_{i,j} |a_{ij}|^2$.

   (Hint: When A was diagonal, we showed this in class. Prove that all the quantities in the above inequality do not change if A is replaced by UAU^{-1} where U is an orthogonal matrix. Then use the fact that any symmetric matrix can be diagonalized).

3. Prove that if the columns of an $m \times n$ matix B are linearly independent, then the $n \times n$ matrix $B^t B$ is invertible.