Due : December 15th, 2018

Instructions: These questions are intended as follow up questions to SWMS- 2018 and the participation in follow up to SWMS-2018[1] will be based on performance in these. Please write down complete solutions to each of the problems. Begin each new problem on a new page and your NAME on every page. Mail the assignments to : `Anita Naolekar and Siva Athreya, Statmath Unit, Indian Statistical Institute, 8th Mile Mysore Road, R.V. College Post, Bangalore 560059.`

[**Calculus**] You may refer to chapters 1-4 from *Calculus* by G. Thomas et. al.

1. In each of the cases below decide if $\{x_n\}_{n\geq 1}$ converges or not:
   i. $x_n = \frac{2^n}{n!}$,
   ii. $x_n = \sqrt{n^2 - n} - n$
   iii. $x_n = nb^n$, for $b \in (0, 1)$.
   iv. $x_n = \frac{n^\alpha}{(1+p)^n}$ with $\alpha \in \mathbb{R}, p > 0$.

$1^\star$. Let[2] $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Let $c \in \mathbb{R}$ with $f'(c) > 0$. Does this imply that f is increasing, on an interval $(c - \delta, c + \delta)$ for some $\delta > 0$?

[**Probability**] You may refer to chapters 1-4 in Probability and Statistics with Examples using R: http://www.isibang.ac.in/∼athreya/psweur/

2. Suppose two teams play a series of games, each producing a winner and a loser, until one team has won two more games than the other. Let G be the total number of games played. Assuming each team has chance 0.5 to win each game, independent of results of the previous games. Find the expected value of G.

[1] to be possibly held in May 2019
[2] Repeated from Assignment 1, as no one got it correct

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[**Statistics**] You may refer to chapters $1-12$ from *Statistics* by Freedman et. all

3. A follow-back study on a large sample of death certificates shows the average age at death is smaller for left-handed people than for right-handers. (In this kind of study, surviving relatives are interviewed.)
   i. Suppose that, other things being equal (age, sex, race, income, etc.), left-handed people are more at risk from accident and disease than right handers. Could that explain a difference in average age at death?
   ii. During the twentieth ccntury, there were big changes in child-rearing practices. In the early part of the century, parents insisted on raising children to be right-handed. By mid-century, parents were much more tolerant of left-handedness. Could that explain a difference in average age at death of left-handed and right-handed people in 2oo5?
   iii. What do you conclude from the death certificate data?

[**Linear Algebra**] You may refer to chapters $1, 2$ and 8 from *Linear Algebra* by A. R Rao and P. Bhimasankaram.

4(a) Define an inner product space.

4(b) Let $(V, <, >)$ be an inner product space and let $\mathcal{B} = \{v_1, v_2, \ldots, v_n\}$ be an ordered basis for V. Describe the Gram-Schmidt procedure to construct an orthonormal basis from $\mathcal{B}$. Illustrate geometrically.

4(c) Let P be the plane in $\mathbb{R}^3$ spanned by the vectors $x_1 = (1, 2, 2)^t$ and $x_2 = (-1, 0, 2)^t$.
(i) Find an orthonormal basis for P.
(ii) Extend it to an orthonormal basis for $\mathbb{R}^3$.

4(d) Let $\mathcal{P}_2(\mathbb{R})$ denote the real vector space of all real polynomials with degrees atmost 2. Consider the inner product on $\mathcal{P}_2(\mathbb{R})$ given by $< p, q >= \int_0^1 p(x)q(x)dx$. Apply the Gram-Schmidt procedure to the basis $\{1, x, x^2\}$ to produce an orthonormal basis of $\mathcal{P}_2(\mathbb{R})$.

[3] to be possibly held in May 2019