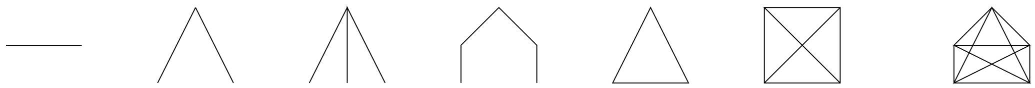
1. Verify that the threshold probability $\hat{p}$ for the random graphs in $G(n,p)$ to contain G given by:



   up to equivalence is $n^{-2}, n^{-\frac{3}{2}}, n^{-\frac{4}{3}}, n^{-\frac{5}{4}}, n^{-1}n^{-\frac{2}{3}}, n^{-\frac{1}{2}}$.

2. Let $\Gamma = [N]$. Let $\Gamma_{p_1,p_2,\ldots p_N}$ be the set where element $i = 1, 2, \ldots, N$ is chosen independent of the others with probability p_i. Let S be a family of non-empty subsets of Γ and for each $A \in S$ let $I_A = 1(A \subset \Gamma_{p_1,\ldots,p_N})$. Let $X = \sum_{A\in S} I_A$.

   (a) Show that $P(X = 0) \geq \exp(-\frac{E(X)}{1-\max_i p_i})$

   (b) Let $\Delta = \frac{1}{2}\sum_{A\neq B, A\cap B\neq\emptyset} E(I_A I_B)$. Then show that $P(X = 0) \leq \exp(-\lambda + \Delta)$ and $P(X = 0) \leq \exp(-\frac{\lambda^2}{\lambda+2\Delta})$

3. Let G be a graph with at least one edge.

   (a) For each $H \subseteq G$ there are $O(n^{2v_G - v_H})$ pairs (G_1, G_2) copies of G in the complete graphs K_n with $G_1 \cap G_2$ isomorphic to H.

   (b) Suppose $\Phi_G = \min(E(X_H) : H \subset G, e_H > 0)$ with X_H being the number of copies of the given graph H in $G(n,p)$ and $\phi_G \to \infty$ then show that $\frac{X_G}{E(X_G)} \xrightarrow{p} 1$

   (c) For every sequence $p \equiv p(n) < 1$, show that $\exp(-\frac{1}{1-p}\Phi_G) \leq P(G(n,p) \not\subset G) \leq \exp(-c_1\phi_G)$, where c_1 is a positive constant.

   (d) If G is a strictly balanced graph(I.e. $\frac{e_H}{v_H} < \frac{e_G}{v_G}$ whenever $H \subset G$ and $np^{m(G))} \to c > 0$ then $X_G \xrightarrow{d}$ Poisson(λ) with $\lambda = \frac{c^{v_G}}{aut(G)}$.

4. **Project 1:** (*Preferred degree distribution*) Person A specifies a degree distribution sequence to you, say $\{d_i : i \geq 1\}$. with expected degree size greater than or equal to 2. Can you construct a *random* graph on n vertices $\mathcal{G}_n$ such that degree of a randomly chosen vertex, D_n, satisfies $P(D_n = i) \to d_i$?

   Suppose $d_1, \ldots d_n$ is given to you and $T = \sum_{k=1}^{n} d_k$ is even. Take T blank cards and write j on d_j cards. Then shuffle the pack thoroughly (7 times!). Deal the cards two at a time. For each pair (l, m) that comes out mark an edge from l to m.

   (a) List out the problems one encounters with the above construction and make the idea mathematically precise.

   (b) Can you prove *a corrected version* for which the produced graph will have the desired limiting degree distribution ?

5. **Project 2:** (*Preferential attachment model*) "Starting with a small number (m_0) of vertices, at every time step we add a new vertex with $m(\leq m_0)$ edges that link the new vertex to m different vertices already present in the system. To incorporate preferential attachment, we assume that the probability Π that a new vertex will be connected to a vertex i depends on the connectivity k_i of that vertex, so that $\Pi(k) = \frac{k_i}{\sum_j k_j}$. After t steps the model leads to a random network with $t + m_0$ vertices and mt edges."

   There are two key issues with the above construction: (a) how to start ? (b) how to choose vertices to attach ?.

   (a) Can you first identify both the issues precisely. Then proceed to provide a mathematical correction of the above idea.

   (b) What is your guess for the limiting degree distribution for this graph ?