1. Let $X \stackrel{d}{=} Bin(n,p)$, $\lambda = np$ and $\phi(x) = (1+x)\ln(1+x) - x$, $x \geq -1$, (and $\phi(x) = \infty$ for $x < -1$). Show that

$$P(X \geq E(X) + t) \leq \exp(-\lambda\phi(\frac{t}{\lambda})) \leq \exp(-\frac{t^2}{2(\lambda + \frac{t}{3})})$$

   and

$$P(X \geq E(X) - t) \leq \exp(-\lambda\phi(\frac{-t}{\lambda})) \leq \exp(-\frac{t^2}{2\lambda})$$

2. Let Γ be any set. Suppose, for any $0 < p < 1$, we construct $\Gamma_p \subset \Gamma$ by choosing each element of Γ independently with probability p. Let $0 < p_1 < 1$ and $0 < p_2 < 1$. Show that $\Gamma_{p_1} \cup \Gamma_{p_2} = \Gamma_p$ for some $0 < p < 1$ and find p in terms of p_1 and p_2.

3. Let Γ be any set. Suppose, for any $0 < M \leq |\Gamma|$, we choose $\Gamma_M \subset \Gamma$ uniformly from all the subsets of Γ of size M. Let $0 < p < 1$, Γ_p be as above and Q be any arbitrary property. Show that $P(\Gamma_M \in Q) = P(\Gamma_p \in Q||\Gamma_p| = m)$.

4. Let Q be an increasing property of subsets of a set $\Gamma \equiv \Gamma(n)$ and $0 \leq M_1 \leq M_2 \leq N = |\Gamma(n)|$. Show that $P(\Gamma_{M_1} \in Q) \leq P(\Gamma_{M_2} \in Q)$.

5. For $0 \leq p \leq 1$. Let Γ and Γ_p be as above. Let Q be an increasing property of subsets. Show that The function $f : [0,1] \to [0,1]$ given by $f(p) = P(\Gamma_p \in Q)$ is strictly increasing and continuous. Hence there

6. Let $0 < p \equiv p(n) < 1$, $\Gamma = \{1, \ldots, n\}$. Let Q be the property of containing a 3-term arithmetic progression. Let Γ_p be set formed choosing each element of Γ independently with probability p. Let X be the number of arithmetic progressions of length 3.
   (a) Show that there are constants c_1, c_2 such that $c_1 n^2 p^3 \leq E(X) \leq c_2 n^2 p^3$.
   (b) Calculate $\mathrm{Var}(X)$.
   (c) Decide for what sequences p does $P(X = 0) \to 0$.
   (d) Can you generalise this argument for arithmetic progression for length k and find a threshold for Q in such cases ?

7. Provide a counter-example to Proposition A, presented in lecture.

8. **Project:** Suppose G is a deterministic graph on n vertices. With each edge G associate two exponential clocks with parameter o and c. At intial time $t = 0$ all edges in G are open. When the o clock ticks an edge becomes open and when the c clock ticks then edge in G becomes closed. At time $t > 0$, one goes and chooses two vertices v, w in G. What is the chance that there is a connected path between v and w ?