# New Directions in Probability

Indian Statistical Institute, Bangalore, May 30 -June 4th, 2013. All talks will be held in the auditorium, 2nd Floor, Main Building.

#### Date : May 30th, 2013

10:00	Anish Sarkar	Some rigourous results in a Modified Bak-Sneppen Model
10:40	Rongfeng Sun	Continuum Limits of the Random Pinning Model Un- der Weak Coupling
11:45	Rohini Kumar	Large deviations in multi-time scale stochastic systems
12:25	Hubert Lacoin	Non-monotonicity for biased random walk on trees without leaves $% \left( $
14:30	Anita Winter	Convergence of bi-measure R-trees and the subtree prune process
15:10	Akira Sakai	Recent progress in the lace expansion

#### Date : May 31st, 2013

10:00	Alexander Drewitz	Asymptotic behavior of the critical parameter for level- set percolation of the Gaussian free field
10:40	Anup Biswas	Law of Large Numbers for Queues under Earliest Dead- line First Scheduling
11:45	Daisuke Shiraishi	Non-intersecting random walks in low dimensions
12:25	Debleena Thacker	Infinite Color Urn Models
14:30	Chiranjib Mukherjee	$Convergence \ of \ path \ measures \ with \ mean-field \ type interactions$
15:10	Partha Dey	Energy Landscape for large average submatrices in Gaussian random matrices

New Directions in Probability Indian Statistical Institute, Bangalore, May 30 -June 4th, 2013.

#### Date : June 1st, 2013

10:00	Codina Cotar	On finite and infinite-bodied optimal transportation with Coulomb cost
10:40	Ryoki Fukushima	Localization of Brownian motion in random potentials
11:45	Ghurumuruhan Ganesan	Extinction probability and total progeny of predator- prey dynamics on infinite trees
12:25	Neeraja Sahasrabudhe	Covariance Realization Problem for Spin Systems
14:30	Omer Angel	Half planar random maps and percolation exponents
15:10	Antar Bandyopadhyay	Nearest Neighbor Algorithm for Mean Field Traveling Salesman Problem

Date : June 2nd, 2013

Off Day

## New Directions in Probability

Indian Statistical Institute, Bangalore, May 30 -June 4th, 2013.

#### Date : June 3rd, 2013

10:00	Manjunath Krishnapur	On zeros of random functions
10:40	Kumarjit Saha	Random directed forest and the Brownian web
11:45	Adrian Roellin	Testing dense graphs for homogeneity
12:25	Makiko Sasada	Mixing rates and hydrodynamics for stochastic energy exchange models with degenerate rate functions
14:30	Arnab Ganguly	A few approaches to large deviations for stochastic dif- ferential equations
15:10	Sreekar Vadlamani	Scaling limits for critical quantum random graphs

#### Date : June 4th, 2013

10:00	Kavita Ramanan	Obliquely reflecting Brownian motion in fractal domains
10:40	Rajat Hazra	Thick points for the Gaussian Free Field in 4 dimensions
11:45	Cristian Giardina	Dual processes and exact correlation functions.
12:25	Tom Alberts	Dimension Spectrum of SLE Boundary Collisions
14:30	Kshitij Khare	Convergence for some multivariate Markov chains with polynomial eigenfunctions.
15:10	Arijit Chakrabarty	Limiting spectral distribution for Wigner matrices with dependent Entries

10:00 Anish Sarkar

Some rigourous results in a Modified Bak-Sneppen Model

#### Abstract

Bak-Sneppen model has been proposed in the Statistical Physics literature as a model of self-organised criticality. However, rigourous mathematical derivation of the self-organised criticality has not yet been achieved. In this talk, I will discuss the results which have been proved rigourously and state other conjectures. Finally, I will show that in a modified model, similar results can be proved using very simple arguments.

10:40	Rongfeng Sun	Continuum	Limits	of th	e Random	Pinning	Model	Un-
		der Weak C	oupling					

#### Abstract

We study random pinning models, where the return time distribution of the underlying renewal process is regularly varying with exponent  $-(1 + \alpha)$  for some  $\alpha > 1/2$ . We show that, if we let the system size tend to infinity, while rescaling the disorder strength and bias accordingly, then the partition functions of the random pinning models have non-degenerate limits that can be expressed in terms of Wiener chaos expansions. For  $\alpha > 1$ , these limits are exponentials of Brownian motions with drift. For  $\alpha \in (1/2, 1)$ , these limits can still be interpreted as partition functions, which we use to construct a family of continuum random pinning models. (Joint work with Francesco Caravenna and Nikos Zygouras)

11:45 Rohini Kumar Large deviations in multi-time scale stochastic systems

#### Abstract

Using the theory of viscosity solutions of partial differential equations, we prove large deviation results for multi-time scale stochastic systems. We initially look at two time scale stochastic systems where the fast and slow processes are diffusions given by coupled SDEs. This problem was motivated from finance when pricing options close to maturity under the assumption of fast mean-reverting stochastic volatility. This was joint work with Jean-Pierre Fouque and Jin Feng. We will also look at possible extensions of these results to a two time scale system where the fast process is a jump-diffusion process.

12:25	Hubert Lacoin	Non-monotonicity	for	biased	random	walk	on	trees
		without leaves						

#### Abstract

We consider a random walk on an infinite rooted tree biased away from the root. We exhibit a family of trees for which the asymptotic speed of the walk is not an increasing function of the bias.

14:30	Anita Winter	Convergence	of	bi-measure	R-trees	and	the	subtree
		prune process	6					

#### Abstract

In 1998 Aldous and Pitman constructed a tree-valued Markov chain by pruning off more and more subtrees above randomly chosen edges of a Galton-Watson tree. More recently Abraham, Delmas and He considered a similar process, where the cut-points are chosen in a degree-dependent way. In the same spirit prunings of continuum trees were studied by various authors. However, so far no precise link between the prunings of discrete and continuum trees has been given.

In this talk we encode trees as metric measure spaces and equip them additionally with a pruning measure, and provide a topology on the space of bi-measure R-trees. We then construct THE subtree prune process and show that convergence of initial states implies convergence of the whole bi-measure valued paths.

(joint work with Wolfgang Lhr and Guillaume Voisin)

15:10 Akira Sakai Recent progress in the lace expansion

#### Abstract

The lace expansion has been a powerful tool to rigorously investigate critical behavior in high dimensions. For example, self-avoiding walk is a statistical-mechanical model for linear polymers and exhibits critical behavior when the fugacity  $\mu$  approaches its critical value  $\mu_c$ :  $\chi_{\mu} \equiv \sum_{\omega:o\to} \mu^{|\omega|} \mathbf{1}_{\{\omega \text{ is SAW}\}} \approx (\mu_c - \mu)^{-\gamma}$ . By the lace expansion initiated by Brydges and Spencer in 1980s and then developed by Hara and Slade in 1990s, it is now known that the susceptibility  $\chi_{\mu}$  for nearest-neighbor SAW above 4 dimensions diverges as a multiple of  $(\mu_c - \mu)^{-1}$ . The key idea is to rewrite  $\mathbf{1}_{\{\omega \text{ is SAW}\}}$  by  $\prod_{0 \leq s < t \leq |\omega|} (1 - \delta_{\omega_s, \omega_t})$  and expand the product in a nicely way so that the self-avoiding constraint is partially restored.

The number of models/problems to which the lace expansion can be applied is getting larger, but it is still limited. The goal of my talk is to extend the range of applicability by considering the following two problems above 4 dimensions.

- Non-intersection probability of two random walks starting from  $o \in \mathbb{Z}^d$ .
- Asymptotic behavior of the critical two-point function for the  $\varphi^4$  model in  $\mathbb{Z}^d$ .

The former is solved by a simple extension of the aforementioned idea for SAW. The latter is solved by approximating the  $\varphi^4$  model by a certain Ising model on  $\mathbb{Z}^d \times \{1, \ldots, N\}$  and then using the lace expansion for the Ising model. I will explain the analysis as long as time permits.

10:00	Alexander Drewitz	Asymptotic behavior of the critical parameter for level-
		set percolation of the Gaussian free field

#### Abstract

We consider the Gaussian free field in  $\mathbb{Z}^d$ ,  $d \ge 3$ . It is known that there exists a non-trivial phase transition for its level set percolation; i.e., there exists a critical parameter  $h_*(d) \in [0, \infty)$  such that for  $h < h_*(d)$  the excursion set above level h does have a unique infinite connected component, whereas for  $h > h_*(d)$  it consists of finite connected components only.

We investigate the asymptotic behavior of  $h_*(d)$  as  $d \to \infty$  and give some ideas on the proof of this asymptotics.

(Joint work with P.-F. Rodriguez)

10:40 Anup Biswas

Law of Large Numbers for Queues under Earliest Deadline First Scheduling

#### Abstract

In recent days, queuing systems with impatient customers have gain considerable interest. In this talk we consider queuing systems in which each arriving customer has a deadline and the customer leaves the system if he/she does not reach the server before his/her deadline elapses. Earliest-Deadline-First (EDF) policy has proven optimal in certain sense for these systems. We start with a single server queuing system working under EDF scheduling policy and establish the LLN limits for the queue measures and reneged customers. We show that the LLN limits of queue lengths are related to a Skorohod reflection problem in a time-varying domain. Extension of these results to other queuing systems will also be discussed.

11:45 Daisuke Shiraishi Non-intersecting random walks in low dimensions

#### Abstract

We consider two random walks conditioned never to intersect in  $\mathbb{Z}^2$ . We show that each of them has infinitely many global cut times with probability one. In fact, we prove that the number of global cut times up to n grows like  $n^{3/8}$ . Next we consider the union of their trajectories to be a random subgraph of  $\mathbb{Z}^2$  and show the subdiffusivity of the simple random walk on this graph.

12:25 Debleena Thacker Infinite Color Urn Models

#### Abstract

In this presentation we introduce a new type of urn model with infinitely but countably many colors of balls. We focus mainly on colors indexed by the *d*-dimensional integer lattice and replacement matrices associated with bounded increment random walks on it. We prove the central and local limit theorems for the expected configuration of the urn and show that irrespective of the transience or recurrence of the underlying random walks, there exists universal centering and scaling of the configuration giving appropriate Gaussian limits.

14:30 Chiranjib Mukherjee Convergence of path measures with mean-field type interactions

#### Abstract

We consider long time behavior of Gibbs measure on three dimensional Brownian paths with mean-field type Hamiltonian with Coulomb interaction. The free energy admits a well-studied variational formula (Donsker-Varadhan [1983], Lieb [1976]). It turns out that the long time asymptotics of the path measures converges to a mixture of the minimizers of the free energy variational formula. This model is related to the Polaron problem, where the behavior of the path measures (in a certain regime known as 'strong coupling') is of interest and possesses a number of open problems. This is joint work (in progress) with Erwin Bolthausen (Zrich)

15:10 Partha Dey Energy Landscape for large average submatrices in Gaussian random matrices

#### Abstract

The problem of finding large average submatrices of a real-valued matrix arises in the exploratory analysis of data from a variety of disciplines, ranging from genomics to social sciences. We provide a detailed asymptotic analysis of large average submatrices of an  $n \times n$  Gaussian random matrix. For fixed k we identify the average and the joint distribution of the  $k \times k$  submatrix with largest average value. As a dual result, we establish that, for any given  $\gamma_t > 0$ , the size of the largest square sub-matrix with average bigger than  $\gamma_t$  is, for large n, equal to one of two consecutive integers near  $4\gamma_t^{-2}(\log n - \log \log n)$ .

We then turn our attention to submatrices with dominant row and column sums, which arise as the local optima of iterative search procedures for large average submatrices. For fixed k, we identify the average and joint distribution of a typical  $k \times k$  submatrices with dominant row and column sums, and we carry out a detailed analysis of the number  $L_n(k)$ of such submatrices, beginning with the mean and variance of  $L_n(k)$  which has a very atypical behavior. In particular, for k = 2 and k = 3, the order of the means are  $o(n^2)$ and  $o(n^3)$ , while the variances are  $n^{8/3}$  and  $n^{9/2}$ , respectively, with logarithmic corrections. Our principal result is a Gaussian central limit theorem for  $L_n(k)$  that is based on a new variant of Stein's method. 10:00 Codina Cotar

 $On \ finite \ and \ infinite-bodied \ optimal \ transportation \\ with \ Coulomb \ cost$ 

#### Abstract

In recent joint work with Gero Friesecke and Claudia Klueppelberg (2011, 2013), we discovered that in a natural scaling limit, the celebrated Hohenberg-Kohn density functional from electronic structure theory reduces to an optimal transport problem. We also proved a transferable main theorem independently of Coulomb/electrons of general interest in optimal transport, namely weak<sup>\*</sup> density of smooth plans with given marginal in the set of all such plans.

In new joint work with Gero Friesecke and Brendan Pass (2013), we find by means of techniques from exchangeable processes in probability theory an explicit solution to the infinite marginals OT problem with Coulomb cost. Moreover, we discuss by a combination of probabilistic and optimal transport methods the answer to the question of uniqueness for the many-bodied optimal transport problem with Coulomb cost.

10:40 Ryoki Fukushima Localization of Brownian motion in random potentials

#### Abstract

We consider the Brownian motion interacting with a random potential. More precisely, we define a probability on the path space by weighting the Wiener measure by the Feynman-Kac functional. Under this measure, the process tends to go to a place where the random potential attains atypically small value. I will present several formulations and results on the path localization.

11:45 Ghurumuruhan Ganesan Convergence rate of locally determinable Poisson functionals

#### Abstract

In this talk, we study convergence rate of locally determinable functionals of Poisson processes. Let  $\mathcal{N}$  denote a realization of a homogenous Poisson point process in  $\mathbb{R}^d$  with intensity  $\lambda$  and place an independent mark  $t_x$  on each point x of  $\mathcal{N}$ . Let  $\mathcal{N}_M$  be the resulting marked process and let  $f(x) = f(x, \mathcal{N}_M), x \in \mathcal{N}$  be a locally determinable function. Letting  $W = [-1/2, 1/2]^d$ , we evaluate the rate of convergence of

$$\frac{1}{n^d}\sum_{x\in nW\cap\mathcal{N}}f(x)$$

to its mean as  $n \to \infty$ , in terms of the decay rate of radius of determinability. We apply our results to determine the rate of convergence of functionals of Poisson Voronoi tessellation and the Poisson Boolean model.

12:25 Neeraja Sahasrabudhe Covariance Realization Problem for Spin Systems

#### Abstract

Deriving the form of the optimal solution of a maximum entropy problem, we obtain an infinite family of linear inequalities characterizing the polytope of spin correlation matrices. For  $n \leq 6$  the facet description of such polytope is provided through a minimal system of Bell-type inequalities.

An explicit method to determine the matrix that parametrizes the maximum entropy measure is described and algorithms to obtain this matrix (consequently the maximum entropy measure) are discussed

14:30 Omer Angel

Half planar random maps and percolation exponents

#### Abstract

Certain half planar random maps are known to have a certain "domain Markov" property. I will describe work ith Nicolas Curien where we exploit this property to derive various exponents associated with critical percolation on the random maps.

15:10	Antar Bandyopadhyay	Nearest 1	Veighbor	Algorithm	for	Mean	Field	Traveling
		Salesman	Problem	ı				

#### Abstract

In this talk we will consider the mean field traveling salesman problem, where the intercity distances are taken to be i.i.d. with some distribution F. This paper focus on the *nearest* neighbor tour which is to move to the nearest non-visited city and we show that if F admits a continuous density at 0 which takes a strictly positive value at 0 then the total length of the nearest neighbor tour, asymptotically almost surely scales as log n. Similar result is known for Euclidean TSP and nearest neighbor tour. We further derive the limiting behavior of the total length of the nearest neighbor tour for a general distribution function F with certain assumptions and show that its asymptotic properties are determined by the scaling properties of the density of F at 0.

10:00 Manjunath Krishnapur On zeros of random functions

#### Abstract

We will present some recent results in the study of zeros of random polynomials and related random functions.

10:40 Kumarjit Saha

Random directed forest and the Brownian web

#### Abstract

Consider the *d* dimensional lattice  $\mathbb{Z}^d$  where each vertex is *open* or *closed* with probability *p* or 1 - p respectively. An open vertex  $\mathbf{u} := (\mathbf{u}(1), \mathbf{u}(2), \ldots, \mathbf{u}(d))$  is connected by an edge to another open vertex which has the minimum  $L_1$  distance among all the open vertices with  $\mathbf{x}(d) > \mathbf{u}(d)$ . It is shown that this random graph is a tree almost surely for d = 2 and 3 and it is an infinite collection of disjoint trees for  $d \ge 4$ . In addition for d = 2, we show that when properly scaled, family of its paths converges in distribution to the Brownian web.

11:45 Adrian Roellin

Testing dense graphs for homogeneity

#### Abstract

A very basic problem and starting point in network analysis is to test whether a given network is compatible with a homogeneous Erds-Renyi random graph model, where two nodes are connected with a fixed probability p, independent of all other connections. Several test statistics have been proposed to tackle this problem: diameter of the biggest component, maximum degree, number of triangles, average path length, clustering coefficient, etc. Whereas (asymptotic) distributions of these statistics under the null hypothesis can and have been calculated, it remains mostly unclear how they perform under alternative models. We propose a new test statistic that is based on the number of edges and number of 4-cycles, which asymptotically can detect any heterogeneity in the edge probabilities in the case of dense graphs, that is, the case when the average degree grows linearly with the number of nodes. The proofs are based on bivariate normal approximation via Stein's method (which will not be discussed in this talk) and dense graph limit theory (which will be briefly introduced). 12:25 Makiko Sasada Mixing rates and hydrodynamics for stochastic energy exchange models with degenerate rate functions

#### Abstract

In recent years, stochastic energy exchange systems of locally confined particles in interaction have been studied intensively, as accessible models for the rigorous study of the derivation of Fouriers law from microscopic dynamics of mechanical origin. As a generalization of them, Alexander Grigo, Konstantin Khanin and Domokos Szsz introduced a class of Markov jump processes of energies. In this talk, I will consider a special class of these processes where the rate function of the energy exchange does not have a uniform lower bound. As pointed out by Grigo and his co-authors, such a rate function naturally appears in the models that originate from mechanical models, but it leads to serious technical complications in the analysis of the spectral gap. I will give a lower bound estimate of the spectral gap and examples belonging to the class. Their macroscopic properties and scaling limits will be also discussed.

14:30 Arnab Ganguly A few approaches to large deviations for stochastic differential equations

#### Abstract

In this talk I will give an overview of some general approaches to study large deviations of stochastic differential equations (SDEs). These methods also extend to the case of infinitedimensional SDEs. Since many Markov processes can be represented as solutions of appropriate SDEs, these methods provide a systematic way to investigate large deviation principle of a large class of Markov processes.

15:10 Sreekar Vadlamani Scaling limits for critical quantum random graphs

#### Abstract

Quantum random graphs, which are a generalization of Erds-Rnyi random graphs, were first introduced by Ioffe and Levit in 2007. We study the behavior of appropriately scaled quantum random graphs inside the "critical window". In particular, we prove that the rescaled sizes of components of quantum random graphs converge in distribution to excursion-lengths of a certain process related to Brownian motion. These results are generalization of Aldous' results on scaling limits of component sizes of Erds-Rnyi random graphs, to the case of quantum random graphs. 10:00 Kavita Ramanan

Obliquely reflecting Brownian motion in fractal domains

#### Abstract

The definition of obliquely reflecting Brownian motion (ORBM) in terms of the reflection vector field on the boundary is not meaningful for domains with rough (fractal) boundaries. We introduce an alternative way to define ORBM in non-smooth (possibly fractal) bounded simply connected planar domains, and show that this definition is in a suitable sense robust. Key tools are a parametrization of ORBMs in the unit disc, which may be of independent interest, and conformal mappings. This is joint work with K. Burdzy, Z.Q. Chen and D. Marshall.

10:40 Rajat Hazra Thick points for the Gaussian Free Field in 4 dimensions

#### Abstract

We study the fractal properties of the thick points of the 4-dimensional massive Gaussian Free Field. We adopt the definition of Gaussian Free Field (GFF) on  $\mathbb{R}^4$  introduced by Chen and Jakobson (2012) viewed as an abstract Wiener space with underlying Hilbert space given by the Sobolev space  $H^2(\mathbb{R}^4)$ . In this talk we show that for  $0 \le a \le 4$  the Hausdorff dimension of the set of *a*-high points is 4 - a. The set of thick points can be related to the support of the Liouville quantum gravity measure in 4 dimensions introduced by Chen and Jakobson (2012). These results can be seen as an extension of the results obtained in 2 dimensions by Hu, Miller and Peres (2010) for thick points of the GFF on a domain. We also review some results on 2-dimensional Liouville quantum gravity measure introduced by Duplantier and Sheffield (2011).

This is a joint work with Alessandra Cipriani.

11:45 Cristian Giardina Dual proc

Dual processes and exact correlation functions.

#### Abstract

Duality is a powerful tool to analyse stochastic dynamics, by which the original process is studied via a simpler one, called a "dual process". In this talk I will review a new method to construct a dual process which I recently introduced in collaboration with J. Kurchan and F. Redig. The method combines insights of probability theory, mathematical physics and the theory of Lie algebra. Applications will be shown in the context of heat conduction models and for multi-type population models. In both case duality yields an exact solution, and explicit expressions for the process correlation functions can be found. 12:25 Tom Alberts

Dimension Spectrum of SLE Boundary Collisions

#### Abstract

In the range  $4 < \kappa < 8$ , the intersection of the Schramm-Loewner Curve (one of the central objects in the theory of 2-D Conformally Invariant Systems) with the boundary of its domain is a random fractal set. After reviewing some previous results on the dimension and measure of this set, I will describe recent joint work with Ilia Binder and Fredrik Viklund that partitions this set of points according to the generalized "angle" at which the curve hits the boundary, and computes the Hausdorff dimension of each partition set. The Hausdorff dimension as a function of the angle is what we call the dimension spectrum.

14:30 Kshitij Khare Convergence for some multivariate Markov chains with polynomial eigenfunctions.

#### Abstract

In this talk, we will present examples of multivariate Markov chains for which the eigenfunctions turn out to be well-known orthogonal polynomials. This knowledge can be used to come up with exact rates of convergence for these Markov chains. The examples include the multivariate normal autoregressive process and simple models in population genetics. Then we will consider some generalizations of the above Markov chains for which the stationary distribution is completely unknown. We derive upper bounds for the total variation distance to stationarity by developing coupling techniques for multivariate state spaces. The talk is based on joint works with Hua Zhou and Nabanita Mukherjee

15:10 Arijit Chakrabarty Limiting spectral distribution for Wigner matrices with dependent Entries

#### Abstract

In this work we show the existence of limiting spectral distribution of a symmetric random matrix whose entries come from a stationary Gaussian process with covariances satisfying a summability condition. We provide an explicit description of the moments of the limiting measure. We also show that in some special cases the Gaussian assumption can be relaxed. The description of the limiting measure can also be made via its Stieltjes transform which is characterized as the solution of a functional equation. In two special cases, we get a description of the limiting measure - one as a free product convolution of two distributions, and the other one as a dilation of the Wigner semicircular law.

This is a joint work with Rajat S. Hazra and Deepayan Sarkar.

### New Directions in Probability

Indian Statistical Institute, Bangalore, May 30 -June 4th, 2013.

#### WIFI Code: ISIbc

#### On: May 30, May 31, June 1, June 3, June 4 2013 :

- 8:30 Breakfast at the Canteen
- 9:30 Coffee 11:20 Snacks and Coffee
- 1:30 Lunch at the Canteen
- 15:50 Snacks and Coffee
- 20:00 Dinner at the Guest house

#### Social Events:

*Date: 1st June 2013:* 

Traditional South Indian dinner on a Banana leaf Venue: Indian Statistical Institute. Time: 8pm Date: 2nd June 2013: Recommended List :

- 1. Bangalore walks (Conference will arrange transport to take you to the walk starting points.)
- 2. Day Trip to Mysore or Belur-Halibed (Trip costs are to be borne by the participants. You can also try KSTDC Package tours. See information for taxi below.

Date: 3rd June 2013: Conference Dinner Restaurant: Kanua, Sarjapur Road. Cuisine: Coastal Karnataka (both vegetarian and non-vegetarian options).

#### Places to eat:

- 1. Online resources: Burrp and Zomato
- 2. Near conference venue: Food court at Gopalan Arcade and The club

#### Phone Numbers:

- 1. Taxi Service (within Bangalore): Fast Track 28889999, Ola Cabs 3355 3355, Meru Cabs 44224422.
- Long distance trips: San Travels 41309777, 9880712267 and the Indian Statistical Institute's travel agent Avion Network, (Ask for Nitin, say from ISI) 26651373, 26651602, 26652226