Due: 1pm August 8th, 2018 and *Problems to be turned in :* 1,2,3

Instructions: Write your name on this sheet. Write down answers on a sheet of paper and each question's answer should begin on a fresh sheet of paper. Staple all the sheets, including this sheet, and submit the same.

1. (**How does the Quadratic Attain is Maximum or Minimum ?**) Let $y = ax^2 + bx + c$. In each of the cases below, draw a *very rough* sketch of y:

   (i) $b^2 - 4ac > 0$ and $a > 0$, (ii) $b^2 - 4ac > 0$ and $a < 0$,

   (iii) $b^2 - 4ac = 0$ and $a < 0$, (iv) $b^2 - 4ac = 0$ and $a > 0$,

   (v) $b^2 - 4ac < 0$ and $a < 0$, (vi) $b^2 - 4ac < 0$ and $a < 0$.
   Distinguish each w.r.t. to y attaining its global maximum or minimum.

2. State Rolle's Theorem.

3. (**Why does the Second Derivative test work ?**) Let $f : \mathbb{R} \to \mathbb{R}$. Suppose f is differentiable two times. Let $d \in \mathbb{R}$ be fixed and $x \in \mathbb{R}, x \neq d$. Let

$$P_1(x) = f(d) + (x - d)f'(d) \text{ and } K = \frac{f(x) - P_1(x)}{(x - d)^2}$$

   (a) Define $F(y) = f(y) - P_1(y) - K(y - d)^2$. Find $F(d)$, $F'(d)$.
   (b) Use Rolle's Theorem to conclude that there is a ξ between d and x such that $F''(\xi) = 0$
   (c) Conclude that for any $d, x \in \mathbb{R}$ $x \neq a$,

$$f(x) = f(d) + (x - d)f'(d) + \frac{(x - d)^2}{2} f''(\xi),$$

   where ξ is a point between d and x.

   (d) **Second Derivative Test:** Suppose $f'(d) = 0$, $f''(d) < 0$. In addition, suppose there is an interval[1] I with $d \in I$ and $f''(z) < 0$ for all $z \in I$.

      i. Let $\xi \in I$. Provide a rough sketch of the graph of $y = f(d) + (x - d)f'(d) + \frac{(x-d)^2}{2} f''(\xi)$.
      ii. Can you conclude if d is a local maximum for f from work done in question 1.
      iii. Then show that f has a local maximum[2]at d.

[1] this is true if the second derivative of f is continuous
[2] We say that f has a local maximum at d if there is an interval I with $d \in I$ and $f(x) \leq f(d)$ for all $x \in d$.