1. Let $a \in \mathbb{R}$ and $\{a_n\}_{n\geq 1}$ be a bounded sequence of real numbers. Consider the following statements :

   (a) There are infinitely many elements of the sequence $\{a_n\}_{n\geq 1}$ inside any interval containing a.

   (b) Inside any interval, I, containing a all but finitely many elements of the sequence $\{a_n\}_{n\geq 1}$ are in I.

   (c) There is a subsequence of $\{a_n\}_{n\geq 1}$ which converges to a.

(i) Rewrite, (a), (b), and (c) in logical notation.
(ii) Decide if they are equivalent statements.

2. Let $\{a_n\}_{n\geq 1}$ be a bounded sequence of real numbers. Consider the following statements :

(a) $\{a_n\}_{n\geq 1}$ converges.

(b) $\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n$

Choose an appropriate method of proof to show $(a) \iff (b)$.