

1. Let $\{a_n\}_{n \geq 1}$ be a bounded sequence of real numbers. Consider the following statements :

- (a) $L = \sup(A)$ where $A = \{x \in \mathbb{R} : x \text{ is a limit point of } a_n\}$.
- (b) For every $\epsilon > 0$:
- there exists $N \geq 1$ such that $a_n < L + \epsilon$ whenever $n \geq N$ and
 - for all $M \geq 1$ there exists $n > M$ such that $a_n > l - \epsilon$.

Choose an appropriate method of proof and show that $(a) \iff (b)$.

2. Let H be a proper subgroup of the additive group \mathbb{Z} . Consider the following statements

(a) $\{18, 30, 40\} \subset H$.

(b) $H = 2\mathbb{Z} := \{2k : k \in \mathbb{Z}\}$

Choose an appropriate method of proof to show $(a) \iff (b)$