Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers.

1. Provide examples of sequences $\{a_n\}$ that satisfy each of the statements below.

   (a) For all $\epsilon > 0$, for all but finitely many $n \in \mathbb{N}$

   $$a_n < 5 + \epsilon \text{ and } a_n > -11 - \epsilon$$

   (b) For all $\epsilon > 0$, there are infinitely many $n \in \mathbb{N}$ such that

   $$\mid a_n - L \mid < \epsilon$$

   for $L = -1, 0, 3$ and $a_n \notin \{-1, 0, 3\}$ for all $n \geq 1$

   (c) For $a > 0$ there are infinitely many $n \in \mathbb{N}$ such that

   $$a_n > a \text{ and}$$

   there are infinitely many $n \in \mathbb{N}$ such that

   $$a_n < -a.$$

2. Write the below statements using logical notation:

   (a) *For every $\epsilon > 0$ there are infinitely many n such that distance of a_n to 0 is less than ϵ*

   (b) *For every $\epsilon > 0$, all but finitely many elements of the sequence a_n are above $11+\epsilon$ and inifinitely many above $11 - \epsilon$*

3. Consider the following statements:

   (a) For every $\epsilon > 0$ there exists $N > 0$ such that $\mid a_n - L \mid < \epsilon$ for all $n > N$.

   (b) There is a $C > 0$ such that for every $\epsilon > 0$ there exists $N > 0$ such that $\mid a_n - L \mid \leq C\epsilon$ for all $n \geq N$.

   (c) For every $N > 0$ there exists $\epsilon > 0$ such that for all $n > N$ implies $\mid a_n - L \mid < \epsilon$.

   (d) There exists $N > 0$ such that for all $\epsilon > 0$ and $n > N$ implies $\mid a_n - L \mid < \epsilon$.

   (e) For every $\epsilon > 0$ and for all $n \geq 1$, there exists $N > 0$ such that $m > N$ implies $\mid a_m - L \mid < \epsilon$.

   (f) For every $\epsilon > 0$ and for all $n \geq 1$, there exists $N > 0$ such that $N > n$ and $\mid a_N - L \mid < \epsilon$.

   Decide which of the above versions are equivalent to the definition of

   $$\lim_{x \to \infty} a_n = L$$

   and which are not. For those that are not equivalent to $\lim_{n\to\infty} a_n = L$ determine, in as simple a language as possible, what they really define. Find examples (if they exist) of sequences that satisfy the definition and of sequences that don't satisfy it.