1. Write the following statements using logical notation and negate them.

   (a) Every classroom has a chair that is not broken.

   Solution:
   In Logical Notation: Let R denote the set of all classrooms, for all $r \in R$ let $C(r)$ denote the chairs in the room r and B denote the set of all broken chairs.

   $$\forall r \in R, \exists c \in C(r) \text{ such that } c \in B.$$

   Negation: $\exists r \in R, \forall c \in C(r)$ we have $c \notin B$. □

   (b) $f : \mathbb{R} \to \mathbb{R}$ is unbounded if for every real number M, some real number x satistfies $\mid f(x) \mid> M$.
   Solution:
   In Logical Notation: $f : \mathbb{R} \to \mathbb{R}$ is unbounded if $\forall M \in R \exists x \in R$ such that $\mid f(x) \mid> M$.
   Negation: $\exists M \in R$ such that $\forall x \in R$ we have $\mid f(x) \mid\leq M$. Such an f is bounded. □

2. Define what is meant by "a sequence $\{a_n\}_{n\geq 1}$".
   Solution: A sequence is a $f : \mathbb{N} \to \mathbb{R}$. We denote $f(n) := a_n$ for aeach $n \geq 1$ and call the function a "a sequence $\{a_n\}$.

3. We say $\lim_{n\to\infty} a_n = L$ if

   For every $\epsilon > 0$ there exists $N > 0$ such that $\mid a_n - L \mid< \epsilon$ whenever $n \geq N$.

   (a) Write a logical statement that says $\lim_{n\to\infty} a_n \neq L$

   Solution: $\exists \epsilon_0 > 0$ such that for all $N > 0$ $\exists n_0 > N$ such that $\mid a_{n_0} - L \mid\geq \epsilon$ □

   (b) Write a logical statement that says $\lim_{n\to\infty} a_n \neq a$ for any $a \in \mathbb{R}$.

   Solution: $\forall a \in \mathbb{R}\, \exists \epsilon_0 > 0$ such that for all $N > 0$ $\exists n_0 > N$ such that $\mid a_{n_0} - R \mid\geq \epsilon$ □

4. If $f : \mathbb{R} \to \mathbb{R}$ is unbounded then construct a sequence $\{a_n\}$ such that $\lim_{n\to\infty} \mid f(a_n) \mid= \infty$

   Solution: Fix an $n \geq 1$. As f is unbounded $\exists x \in \mathbb{R}$ such that $\mid f(x) \mid> n$. Since $n \geq 1$ was arbitary denoting x obtained for each n by a_n we have a sequence $f(a_n)$ with $\mid f(a_n) \mid> n$ for all $n \geq 1$. Therefore,

   $$\text{for any } M > 0 \text{ and for all } n \geq M + 1 \text{ we have } \mid f(a_n) \mid> M.$$

   Therefore $\lim_{n\to\infty} \mid f(a_n) \mid= \infty$. □